KTH Matematik

SF2842: Geometric Control Theory

Homework 2

Due March 2, 16:50pm, 2016

You may use min(5,(your score)/4) as bonus credit on the exam

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1. Consider the system

$$\dot{x} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & a \\ 1 & 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} x,$$

where a is a constant.

(a) For what a does the system have relative degree? [1p]

Solution: For $a \neq 1$ the system has a relative degree $(2, 1)$.

(b) When the system has relative degree, comvert the system into the normal form. [3p]

Solution: Let $\xi_1^1 = x_1$, $\xi_1^2 = x_4$, $\xi_2^1 = x_2 + x_3$, and $z = ax_2 + x_3 - ax_4$. The system can be transformed into its normal form:

$$\begin{aligned} \dot{z} &= -\frac{a}{a-1} z + (1-a)\xi_1^1 - \frac{a^2}{a-1}\xi_1^2 + \frac{2a - a^2}{a-1}\xi_2^1 \\ \dot{\xi}_1^1 &= \xi_1^2 \\ \dot{\xi}_1^2 &= \xi_2^1 + u_1 + u_2 \\ \dot{\xi}_2^1 &= -\frac{1}{a-1} z - \frac{a}{a-1}\xi_1^2 + \frac{1}{a-1}\xi_2^1 + u_1 + au_2 \\ y_1 &= \xi_1^1 \\ y_2 &= \xi_2^1. \end{aligned}$$

(c) Use the Rosenbrock matrix to verify your computation of the transmission zeros from (b). [3p]

Solution: The system has a transmission zero $-\frac{a}{a-1}$ by checking the system matrix, when $a \neq 1$.

2. Consider the system

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= -x_1 - 3x_2 - 3x_3 + u \\
\dot{w}_1 &= w_2 \\
\dot{w}_2 &= -w_1 \\
u &= w_1 \\
y &= c_1 x_1 + c_2 x_2 + x_3,
\end{aligned}$$

where $c_1,\ c_2$ are constant and $c_1 - c_2 + 1 \neq 0$.

(a) Compute the invariant subspace $x = \Pi w$. [2p]

Solution: $\Pi = \frac{1}{4}\begin{pmatrix} -1 & -1 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$.

(b) For what value(s) of c_1, c_2 is the above system (consisting of x and w) unobservable? Explain why. [2p]

Solution: The original system with x is always observable since $c_1 - c_2 + 1 \neq 0$. Only when $c_1 = 1,\ c_2 = 0$, the eigenvalue of S coincides with the transmission zero of the (A, B, C) system, which makes the big system unobservable.

(c) Design c_1, c_2 such that $y(t) = u(t)$ in the steady state. [2p]

Solution: $c_1 = -1,\ c_2 = 2$.

3. Consider:

$$\begin{aligned}
\dot{x}_1 &= x_2 + x_4 \\
\dot{x}_2 &= x_2 + u_1 \\
\dot{x}_3 &= -2x_3 + w_3 + u_2 \\
\dot{x}_4 &= x_1 - \alpha x_3 - x_4 + u_2 \\
\dot{w}_1 &= w_2 \\
\dot{w}_2 &= -w_1 \\
\dot{w}_3 &= 0 \\
e_1 &= x_1 - 2w_1 \\
e_2 &= x_4 - 3w_2
\end{aligned}$$

(a) For what α is the full information output regulation problem solvable? [2p]

Solution: When $\alpha \neq 2$.

(b) For what α is the error feedback output regulation solvable? [2p]

Solution: When $\alpha \neq 2$. Note that when $\alpha = 0$, the pair $\left(\begin{pmatrix} C & -Q \end{pmatrix}, \begin{pmatrix} A & P \\ 0 & S \end{pmatrix} \right)$ is not detectable. However, the error feedback output regulation is still achievable due to the structure of the system.

(c) For $\alpha = 1$, solve the the full information output regulation problem. [3p]

Solution: The feedback control is $u = \Gamma w + K(x - \Pi w)$, where

$$\Gamma = \begin{pmatrix} 1 & 1 & 0 \\ -6 & 7 & 1 \end{pmatrix}, \text{ and } \Pi = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 4 & 1 \\ 0 & 3 & 0 \end{pmatrix}.$$

and K is chosen such that $A + BK$ is stable.