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Robust Control with Classical Methods – QFT

Per-Olof Gutman

- Review of the classical Bode-Nichols control problem
- QFT in the basic Single Input Single Output (SISO) case
- Fundamental Design Limitations
- Identification of Uncertain Transfer functions
- QFT for non-minimum phase and computer controlled systems
- QFT for cascaded systems, and for a class of non-linear plants
- QFT for Multi-Input Multi-Output (MIMO) plants
- A comparison between QFT and other robust and adaptive control

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Identification of Uncertain Transfer Functions

- **The problem**
- **Model building**
- **Frequency response measurements**
  - **- Linear systems**
  - **- Sensor noise, uncertainty, non-linearities**
  - **- The Lissajou figure**
  - **- The Lissajou figure set**
  - **- Averaging methods: the Fourier integral**
  - **- FFT, etfe**
  - **- Goodness of estimate**
  - **- Coherence**
- **Identification from frequency function estimates**
  - **- Manual**
  - **- Non-linear least squares in Bode diagram**
- **Least Squares**
- **Identification in closed loop**

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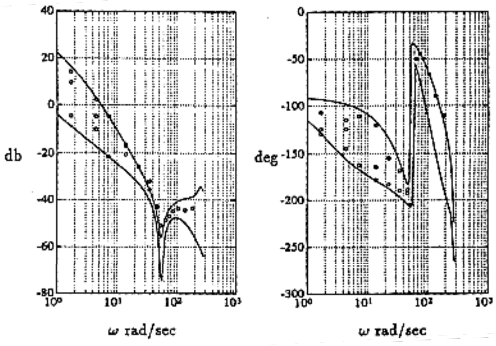
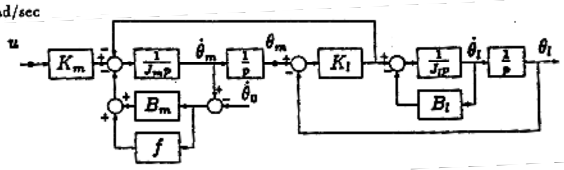
The problem



- **Frequency response experiment on system with friction**



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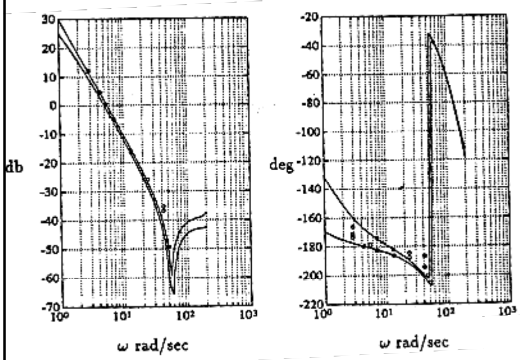
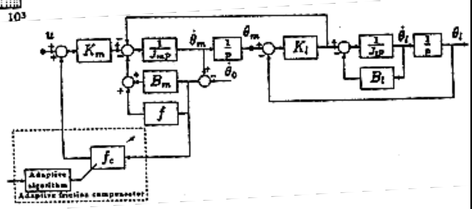
The problem



- **Frequency response experiment on system with friction, and friction compensation (after convergence)**



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The problem

- Different transfer function models may result from
  - non-linearities
  - different operating points
  - noise and disturbances
- We are looking for a set of transfer functions, $\{P_i(s)\}$, representing the plant under all relevant operating conditions, on which to base the robust design.

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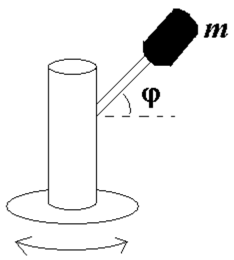
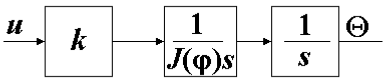
Model building





- Example: Robot arm
- If φ and/or *m* are not measurable, then the moment of inertia is uncertain: $J_{min} \leq J \leq J_{max}$
- ⇒ uncertain transfer function model

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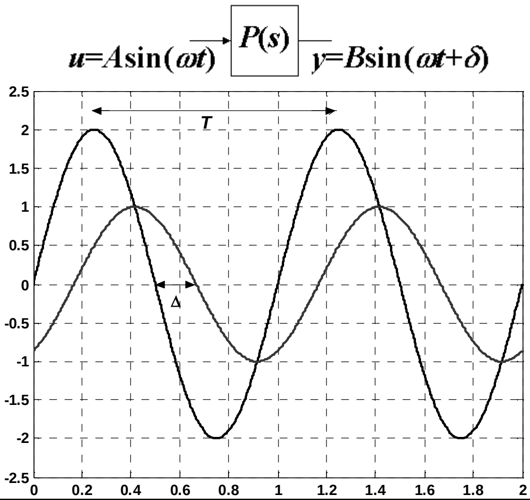
Frequency response measurement

- **Linear systems:**
- $B/A = |P(j\omega)|$
  $\delta = \arg(P(j\omega))$
- $\omega = 2\pi/T$
  $2\pi\Delta/T = |\delta|$



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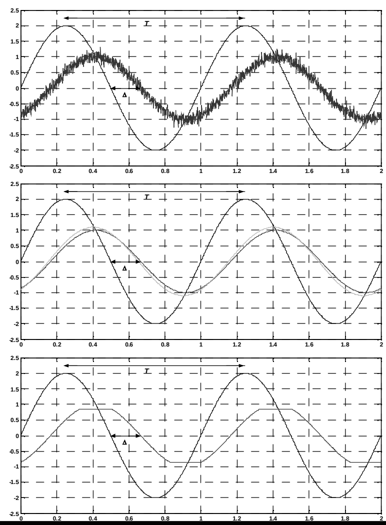
Frequency response measurement, cont'd

- **Direct measurement on oscilloscope difficult due to**
- **sensor noise**
- **uncertainty**
- **non-linearities**



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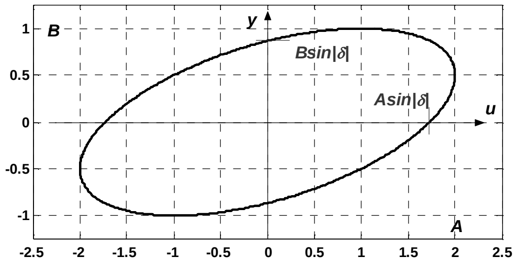
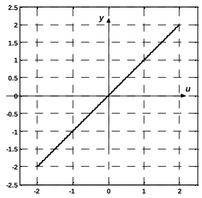
The Lissajou figure

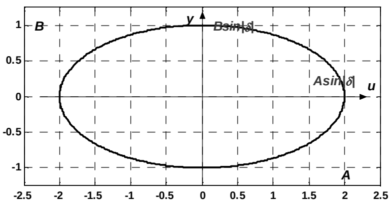
- $u = A\sin(\omega t)$
  $y = B\sin(\omega t + \delta)$



- $A = B,\ \delta = 0$



- $A = 2B,$
  $\delta = -90^{\circ}$



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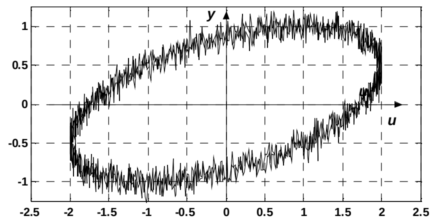
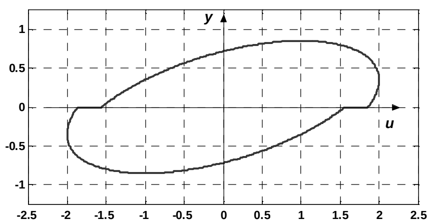
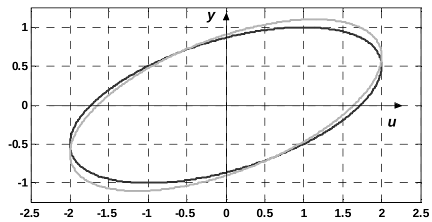
The Lissajou figure, cont'd

- **noise**
- **uncertainty**
- **non-linearity**







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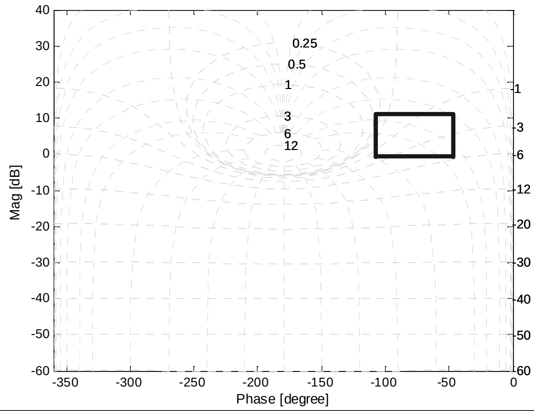
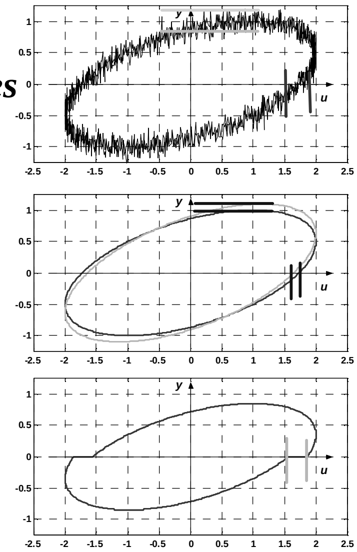
Lissajou figure sets

- **Envelope ⇒ *rectangular templates* with phase and gain extent**





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Lissajou figure sets, cont’d

- **You can do better: Noam Galperin: Modelling and identification of uncertain systems using Lissajou figures. Ph.D.-thesis, Technion, 1997.**
- **Lissajou figure set makes it possible to identify the *convex hull* of the template that gave rise to the Lissajou figure set**

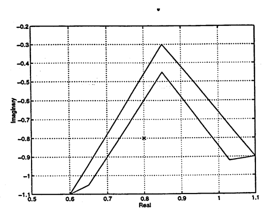


Figure 2.5: A V-shaped value set in the complex plane. The point $G_0 = 0.8 - 0.8j$ is denoted by ×.

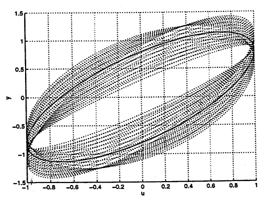


Figure 2.6: The CLFS corresponding to the V-shaped value set in Figure 2.5. The ellipse associated with $G_0 = 0.8 - 0.8j$ is shown with a solid line.

Lissajou figure sets, cont'd

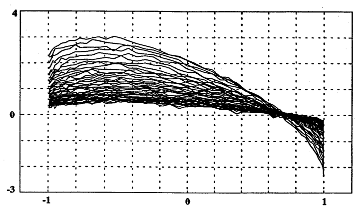
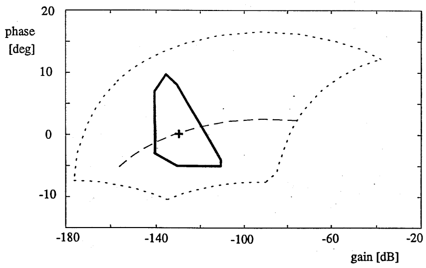


Figure 2.12: The measured LFS of the closed loop electromechanical system.

- **Example: Electro-mechanical system from slide 3.**



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Lissajou figure sets, cont'd

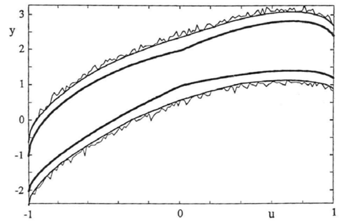

Figure 3.12: The noise free LFS E_0 (thick line), The noisy measured LFS E_m (thin jagged line), and the 1st-stage filtered LFS E_f^1, for Example 6.

- **The *grey level* represents the number of times a given pixel has been traversed by the Lissajou trajectory.**
- **Noam's method can also be used to *filter out noise*, by utilizing the *grey levels* of the Lissajou figure set.**

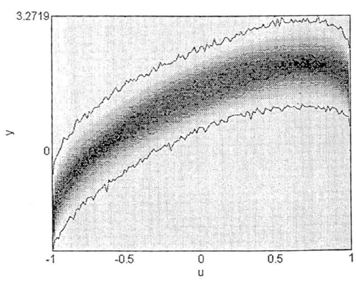

Figure 3.13: Gray level image of the measured histogram h_m for Example 6.

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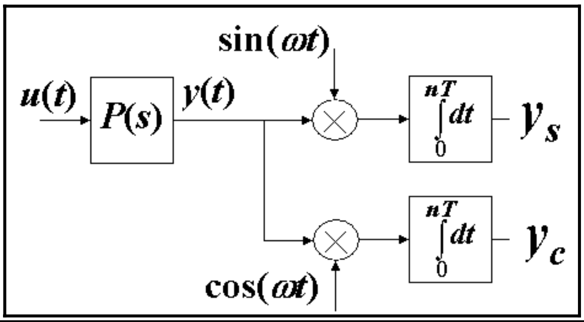
Averaging methods

- ***Fourier Integral Method*** **or** ***Correlation Method*****. Ref. Ljung (1987), Söderström (1987)**
- **$u = A\sin(\omega t)$ $y = B\sin(\omega t+\delta) + e(t)$**



- $\omega = 2\pi/T$
- **This equals FFT at one frequency.**
- **”swept sine”**
- **HP3562, etc.**

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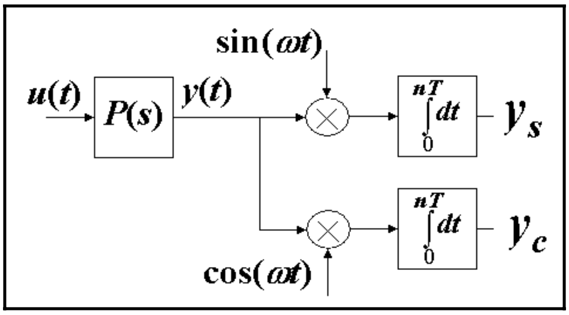
Fourier Integral Method, cont’d

- $y_s = \frac{1}{nT}\int_0^{nT} y(t)\sin(\omega t)dt = \frac{1}{nT}\int_0^{nT} B\sin(\omega t+\delta)\sin(\omega t)dt + \underbrace{\frac{1}{nT}\int_0^{nT} e(t)\sin(\omega t)dt}_{\approx 0,\text{if } e(t) \text{ uncorr. with } \sin(\omega t)} = \frac{B}{2}\cos(\delta) - \underbrace{\frac{B}{2nT}\int_0^{nT}\cos(2\omega t+\delta)dt}_{=0}$
- $y_c = \frac{1}{nT}\int_0^{nT} y(t)\cos(\omega t)dt = \frac{B}{2}\sin(\delta)$



- $\delta = \arctan(y_c/y_s)$
- $B = 2\sqrt{y_c^2 + y_s^2}$
- $\arg(P(j\omega)) = \delta$
- $|P(j\omega)| = B/A$

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FFT, `etfe`

- $Y(s)=P(s)U(s)$, $Y(s)=\mathcal{L}\{y(t)\}$, $U(s)=\mathcal{L}\{u(t)\}$
- $Y(j\omega)= P(j\omega)U(j\omega)$
- $Y(j\omega)=\int_0^\infty y(t)e^{-j\omega t}dt=\int_0^T y(t)e^{-j\omega t}dt$ **, if y(t)≠0 only on [0,T]**

  In experiments, *compact support* is always the case
- $\hat{P}(j\omega)=\mathcal{F}\{y(t)\}/\mathcal{F}\{u(t)\}=$ `fft(y)/fft(u)`=`etfe([y,u])`

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Goodness of estimate

- **The *Power Spectral Density* of $y(t)$ is**
  $\Phi_y(\omega) = Y(j\omega)Y(-j\omega)=|Y(j\omega)|^2$
- **The *Cross Spectral Density* between $u(t)$ and $y(t)$ is**
  $\Phi_{yu}(\omega) = Y(j\omega)U(-j\omega)$
- ***Coherence*:** $\mathcal{K}(\omega)=\sqrt{|\Phi_{yu}(\omega)|^2/(\Phi_y(\omega)\Phi_u(\omega))}$
- **If $Y(s) = P(s)U(s)$, then $\Phi_{yu}(\omega) = P(j\omega)\Phi_u(\omega)$ and**
  $\Phi_y(\omega) = |P(j\omega)|^2 \Phi_u(\omega) = P(-j\omega)\Phi_{yu}(\omega)$**, and** $\kappa(\omega)=1$.
- **Note that** $\hat{P}(j\omega)=\mathcal{F}\{y(t)\}/\mathcal{F}\{u(t)\}= \Phi_{yu}(\omega)/\Phi_u(\omega)$

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Goodness of estimate, cont'd

- **Assume that $Y(s) = P(s)U(s)+E(s)$, and that $e(t)$ is uncorrelated with $u(t)$, i.e. $\Phi_{eu}(\omega)=0$. Then**
- **$\Phi_y(\omega) = |P(j\omega)|^2 \Phi_u(\omega) + \Phi_e(\omega)$ and**
  $\Phi_{yu}(\omega) = P(j\omega)\Phi_u(\omega)$
- $$\mathcal{K}(\omega) = \sqrt{\left|P(j\omega)\Phi_u(\omega)\right|^2 \Big/ \left(\left(\left|P(j\omega)\right|^2 \Phi_u(\omega) + \Phi_e(\omega)\right)\Phi_u(\omega)\right)} < 1$$
- **It can be shown that** $$\operatorname{var}\hat{P}(j\omega) = \Phi_e(\omega) / \Phi_u(\omega)$$

  noise/signal ratio
- **Gives probabilistic value sets!**

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Model identification from frequency function estimates

- **Manual fitting, see Qsyn manual for an example. Command `pmodel` in Qsyn.**
- **Non-linear Least Squares in Bode diagram. Reference: Sidman *et* al., IEEE Transactions on Automatic Control, vol. 36, Sept. 1991, 1065-1070.**
- **Ilan Cohen, Servotronix.**
- **Oded Yaniv, Tel Aviv University.**

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Least Squares

- Assume that the true plant is given as a discrete time t.f. model $y(t) = -a_1 y(t-1) - a_2 y(t-2) - \ldots - a_n y(t-n) + b_0 u(t) + b_1 u(t-1) + \ldots + \; + b_n u(t-n) + e(t)$, with $e(t)$ uncorrelated with $u(t)$, … and $y(t-1)$, ….
- $y(t) = \varphi^T \Theta + e(t)$
  Regressor: $\varphi^T(t) = [-y(t-1) \;\; -y(t-2) \; \ldots \; -y(t-n) \;\; u(t) \;\; u(t-1) \; \ldots \; u(t-n)]$
  Parameter vector: $\Theta = [a_0 \; a_1 \ldots a_n \;\; b_0 \;\; b_1 \ldots \; b_n]^T$ Θ
- Parameter vector estimate $\hat{\Theta}$
- Model error, or *prediction error*: $\varepsilon(t) = (y(t) - \varphi^T \;)$
- *Value function*, or criterion: $V = \sum_{t=1}^{N} \varepsilon^2(t) / (2N)$
- Find $\hat{\Theta}$ such that $V \longrightarrow$ minimum

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Least Squares, cont'd

- Let $Y = [y(1) \;\; y(2) \; \ldots \; y(N)]^T$, $\Phi = [\varphi^T(1) \; \varphi^T(2) \ldots \varphi^T(N)]^T$
- $Y = \Phi\,\Theta + E$
  $\Phi^T Y = \Phi^T \Phi\,\Theta + \Phi^T E \approx \Phi^T \Phi\,\Theta$
- Assume that $R(N) = (\Phi^T \Phi)/N$ has an inverse: *persistant excitation*
- $\hat{\Theta}_N = (\Phi^T \Phi)^{-1} \Phi^T Y$ *pseudo-inverse*
- It can be shown that var $\hat{\Theta}_N \approx \text{var}(e(t)) R^{-1}(N)/N$
- Gives *probabilistic value sets*

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Least Squares, cont'd

- **Recursive Least Squares, also in Kalman filter form, see Ljung(1987), Ljung and Söderström (1985).**
- **Hard least squares bounds, giving deterministic templates, see e.g. Gutman P-O: "On-line parameter interval estimation using Recursive Least Squares".** ***International J. Adaptive Control & Signal Processing*****, vol 8, 61-72, 1994.**

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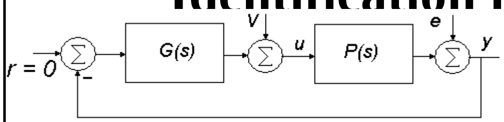
Identification in closed loop



- $Y = \frac{P}{1+PG}V + \frac{1}{1+PG}E \quad (1)$
- $U = \frac{1}{1+PG}V - \frac{G}{1+PG}E \quad (2)$
- $\frac{Y}{U} = \frac{PR+E}{R+GE} \rightarrow \begin{cases} P, \text{ if } R >> E \\ 1/G, \text{ if } R << E \end{cases}$

- **Good signal/noise ratio important, when identifying**
  - **in open loop, w/o feedback**
  - **within closed loop, *Y/U***
  - **in closed loop, *Y/V* and *U/V*, from which P is computed.**
- **Note that in closed loop, frequency separation between sensitivity and compl. sensitivity does not compensate bad s/n ratio.**