TECHNION
Israel Institute of Technology

Robust Control with Classical Methods – QFT

Per-Olof Gutman

- Review of the classical Bode-Nichols control problem
- QFT in the basic Single Input Single Output (SISO) case
- Uncertainty and Fundamental Design Limitations
- QFT for non-minimum phase and computer controlled systems
- QFT for cascaded systems, and for a class of non-linear plants
- QFT for Multi-Input Multi-Output (MIMO) plants
- A comparison between QFT and other robust and adaptive control

TECHNION
Israel Institute of Technology

A comparison between QFT and other robust and adaptive control, I

- **F.N. Bailey and C.H. Hui: Loop Gain-Phase shaping for Single-Input-Single-Output Controller, *IEEE Control Systems Magazine*, 93-101, Jan. 1991.**
- **Loop gain shaping, i.e. LQG/LTR, H_∞, etc**
- **Constraints on the open loop**
- **Example 1**
- **Example 2**
- **Conclusions**

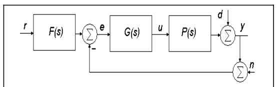
TECHNION
Israel Institute of Technology

Loop gain shaping, e.g. LQG/LTR, H_∞

- Unstructured, additive or multiplicative, plant uncertainty:
  $P(s)=P_0(s)+\Delta_a(s)$ or $P(s)=P_0(s)(1+\Delta_m(s))$
- Low frequency constraint on the open loop, $B_l(\omega)$, is derived from performance and sensitivity specifications.
- High frequency constraint on the open loop, $B_h(\omega)$, is derived from robust stability and/or noise rejection specifications. ... Recall

TECHNION
Israel Institute of Technology

Recall: Sensitivity & complementary sensitivity

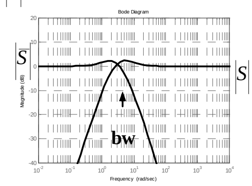


$$Y=\boxed{\frac{1}{1+PG}}D+\frac{FPG}{1+PG}R-\boxed{\frac{PG}{1+PG}}N$$

$$Y=SD+F\bar{S}R-\bar{S}N,\quad \boxed{S+\bar{S}=1}$$

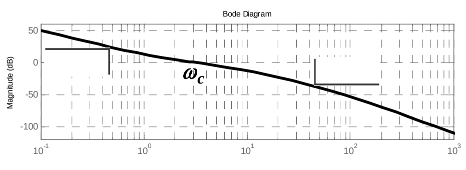
- Desired: $|S|$ small **and** $|\bar{S}|$ small

- Resolution: frequency separation
  - $|S|$ small for low frequencies
  - $|\bar{S}|$ small for high frequencies



TECHNION
Israel Institute of Technology

Recall: Sensitivity and complementary sensitivity



TECHNION
Israel Institute of Technology

Constraints on the open loop

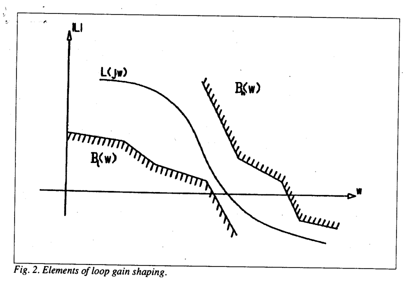


Fig. 2. Elements of loop gain shaping.

Example 1: DC-motor

- DC-motor: $\boldsymbol{P}(\boldsymbol{s})=\dfrac{\boldsymbol{K}}{\left(\boldsymbol{LJ_m}+\boldsymbol{LJ_L}\right)\boldsymbol{s}^2+\left(\boldsymbol{RJ_m}+\boldsymbol{RJ_L}\right)\boldsymbol{s}+\boldsymbol{K}^2}$,

$\boldsymbol{L}=0.0022,\ \boldsymbol{K}=0.2,\ \boldsymbol{R}=0.4,\ \boldsymbol{J_m}=0.0014,\ \boldsymbol{J_L}\in\boldsymbol{J_m}\left[0.05,10\right]$

- Servo specification:

ω [Hz]	<10	[10,100]	300	>1000
$\lvert T(j\omega)\rvert$ [dB]	0±0.05	0±0.2	-20±2	-40±20

- Margin specification: $\left|\boldsymbol{S}\right|<5\text{dB}$

- $B_l(\omega)=\lvert(P-P_0)T_0/(P_0(T-T_0))\rvert$

TECHNION
Israel Institute of Technology

Example 1: unstructured uncertainty

- Unstructured uncertainty approximation for Loop Gain Shaping: $P(s) = P_0(s)(1+ D_m(s)) \Rightarrow$
- $\lvert L_0(j\omega)\rvert<1/\lvert D_m(j\omega)\rvert$ as the robust stability constraint for high ω

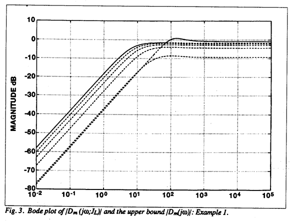


Fig. 3. Bode plot of $\lvert D_m(j\omega;J_L)\rvert$ and the upper bound $\lvert D_m(j\omega)\rvert$: Example 1.

Qsyn – *the* toolbox for robust control systems design P-O Gutman

TECHNION
Israel Institute of Technology

Example 1: LGS-design

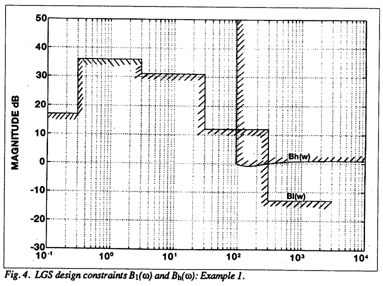


Fig. 4. LGS design constraints $B_l(\omega)$ and $B_h(\omega)$: Example 1.

Qsyn – *the* toolbox for robust control systems design P-O Gutman

TECHNION
Israel Institute of Technology

Example 1: QFT-design

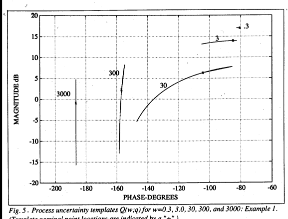


Fig. 5. Process uncertainty templates Q(w;q) for w=0.3, 3.0, 30, 300, and 3000: Example 1. (Template nominal point locations are indicated by a "+".)

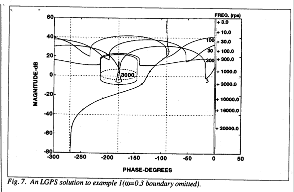


Fig. 7. An LGPS solution to example 1(ω=0.3 boundary omitted).

Qsyn – *the* toolbox for robust control systems design P-O Gutman

TECHNION
Israel Institute of Technology

Example 2: Process with gain uncertainty

$\boldsymbol{P}(\boldsymbol{s})=\dfrac{\boldsymbol{K}}{\boldsymbol{s}\left(\boldsymbol{s}+10\right)},\ \boldsymbol{K}\in\left[9,11\right]$

Servo specification:

ω [Hz]	[0,1]	[1,10]	[10,30]	[30,100]	>100
$\lvert T(j\omega)\rvert$ [dB]	0±0.05	0±0.2	-12±14	-28±14	-40±15

- Margin specification: $\left|\boldsymbol{S}\right|<5\text{dB}$

- $B_l(\omega)=\lvert(P-P_0)T_0/(P_0(T-T_0))\rvert$

Qsyn – *the* toolbox for robust control systems design P-O Gutman

TECHNION
Israel Institute of Technology

Example 2: LGS

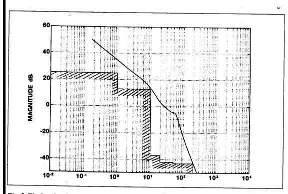


Fig. 8. The low frequency constraint Bl(ω): Example 2.

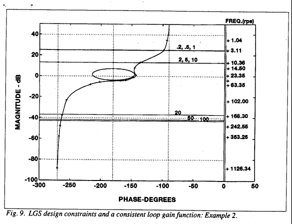


Fig. 9. LGS design constraints and a consistent loop gain function: Example 2.

Qsyn – *the* toolbox for robust control systems design P-O Gutman

Example 2: QFT

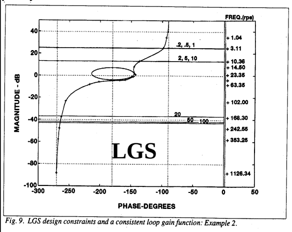


Fig. 9. LGS design constraints and a consistent loop gain function: Example 2.

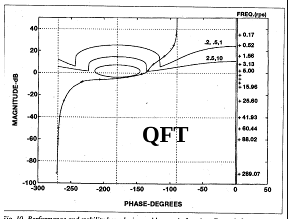


Fig. 10. Performance and stability boundaries and loop gain function: Example 2.

Example 2: Comparison

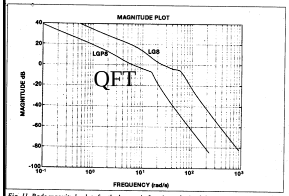


Fig. 11. Bode magnitude plots for the loop gain functions obtained through LGS and LGPS: Example 2.

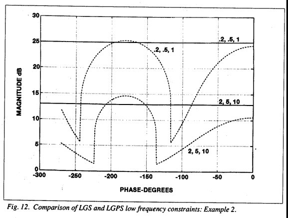


Fig. 12. Comparison of LGS and LGPS low frequency constraints: Example 2.

Conclusion: Robust Control

- Uncertainty description
- Specifications
- Other robust methods ...

A comparison between QFT and other robust and adaptive control, II

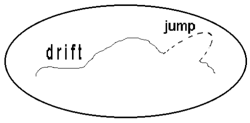
- P-O Gutman, Robust and Adaptive Control - Fidelity or an Open Relationship? *Systems & Control Letters*, vol 49, pp. 9-19, 2003.
  - Abstract
  - Robust Control
  - Adaptive Control
  - Example
  - Exercise
  - Robust vs. Adaptive
  - The rôle of adaptation
  - Adaptive Robust Control
  - Conclusion

Abstract

Robust and adaptive control are essentially meant to **solve the same control problem**: Given an uncertain LTI model set with the assumption that the controlled plant **slowly drifts or occasionally jumps** in the allowed model set, find a controller that satisfies the given servo and disturbance rejection specifications. Specifications on the transient response to a sudden plant change or "plant jump" are easily incorporated into the robust control problem, and if a solution is found, the robust control system does indeed exhibit satisfactory transients to plant jumps. **The reason to use adaptive control is its ability, when the plant does not jump, to maintain the given specifications with a lower-gain control action (or to achieve tighter specifications), and also to solve the control problem for a larger uncertainty set than a robust controller**. Certainly Equivalence based adaptive controllers, however, often exhibit **insufficient robustness** and unsatisfactory transients to plant jumps. It is therefore **suggested in this paper that adaptive control always be built on top of a robust controller in order to marry the advantages of robust and adaptive control**. The concept is called **Adaptive Robust Control**. It may be compared with Gain Scheduling, Two-Time Scale Adaptive Control, Intermittent Adaptive Control, Repeated Auto-Tuning, or Switched Adaptive Control, with the important difference that **the control is switched between robust controllers that are based on plant uncertainty sets that take into account not only the currently estimated plant model set but also the possible jumps and drifts that may occur until the earliest next time the controller can be updated**.

Problem statement

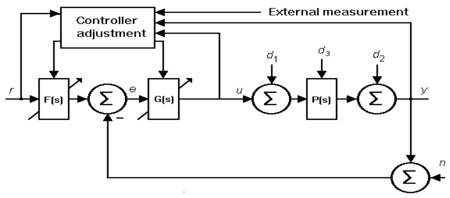
- **Drift:** $P(s)$ **drifts "slowly" among** $\{P_i(s)\}$
  - wear
  - operating point
  - outside environment
- **Jump:** $P(s)$ **jumps "occasionally" within** $\{P_i(s)\}$
  - change of plant equipment
  - partial failure
  - robot arm picks up unknown load
  - switching on
- **"product of plant change and time is small"**



Robust control

- **Include disturbance rejection specifications equivalent to the envisaged "occasional jump".**
- **If a solution is found, then specs are also satisfied for "slow drift"**

Adaptive Control System



Adaptive control

- **Dual control**
  Control signal optimal for estimation and control
  Computationally prohibitive
- **Desired capability for LTI based adaptive control**
  Estimate current process model
  Redesign the controller
  Decide when to estimate/redesign
- "An adaptive controller is a controller with adjustable parameters, and a mechanism for adjusting"
- **Parameters are adjusted such that after convergence, the specs are satisfied**

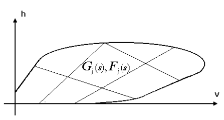
Adaptive control methods

Certainty Equivalence (CE)
- Parameter estimation (RLS, LMS, ...) combined with
  Control design (pole placement, MRAS, ...), directly or indirectly
- Under **ideal conditions (**no noise, no disturbances, no undermodelling, min phase) MRAS gives boundedness and convergence
- **Detuning from CE to handle some non-idealities** (dead-zone, σ–modification, back-stepping, etc)
  - **"Slow drift" OK**
  - **"Occasional jump" ?**
  - **Mixed industrial record**

Auto-tuning
Supervised automatic tuning
- **"Slow drift" ?**
- **"Occasional jump" ?**

Adaptive control methods, cont'd

Gain scheduling
- Pre-computed controllers for each operating condition
  - Separate identification loop
  - Soft transfer between regimes
  - LTI at (almost) all times → no stability problem
  - **"Slow drift" OK**
  - **"Occasional jump" Yes, some**



Two time scale adaptive control
Intermittent adaptive control
Switched adaptive control
- Identification and control loops separated
- LTI at (almost) all times → no stability problem
- **"Slow drift" ?**
- **"Occasional jump" ?**

Example

$$P(s)=\frac{k}{(1+Ts)^2} \quad k\in[1,4] \quad T\in[0.5,2]$$

- Closed loop servo spec for QFT design: ω_B = 2 rad/s

$$G(s)=4\cdot 10^7\cdot\frac{s+0.25}{s}\cdot\frac{s+1.5}{s+30}\cdot\frac{1}{s^2+500s+250000}$$

$$F(s)=2.89\cdot\frac{1}{s^2+1.87s+2.89}$$

- **Closed loop servo spec for Explicit 2nd order adaptive pole placement, with RLS: ω_0 = 1.5 rad/s; ζ = 0.707**

Sampling interval = 0.3 seconds

Example, cont'd

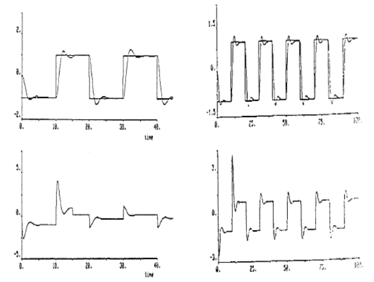


Fig. 4. Simulation of the robust control system. The upper and lower graphs show step responses and the control signals, respectively. On the left, the plant gain k changes from 1 to 4 at time $t = 15$ s while T=1. On the right, the time constant T changes from 1 to 0.5 at time $t = 15$ s while $k = 1$.

Example, cont'd

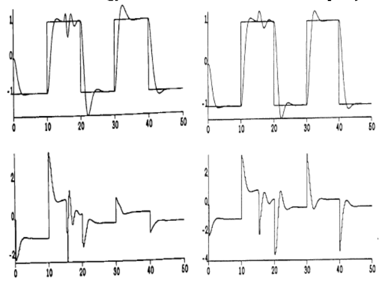


Fig. 5. Simulation of the adaptive control system. The upper and lower graphs show step responses and the control signals, respectively. On the left, the plant gain k changes from 1 to 4 at time $t = 15$ s while T=1. On the right, the time constant T changes from 1 to 0.5 at time $t = 15$ s while $k = 1$.

Exercise

- $P(s)=\frac{ke^{-\tau s}}{s}$
  $k\in[1,3],\ \tau\in[0,1]$
  $P_{\text{nom}}(s)=\frac{3}{s}$
- Spec: $|S|<6$dB max. ω_c

- Design a (i) robust controller, and (ii) an adaptive controller, e.g. CE pole placement.
- Compare plant input disturbance rejection response!

Robust vs. Adaptive

	Robust	CE Adaptive
Stability	√	restrictive assumptions
"occasional jump"	√	√
"slow drift"	√	√√√

√√√ **Adaptive solution** can be found for **larger plant uncertainty** and/or **tighter specifications**

- **Challenge**: How to marry Robust and Adaptive to get the best of both?

Trade-off in feedback design

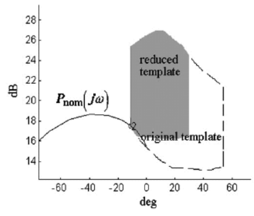


Fig. 8. Original template, $\{P(2j)\}$, from (7); the nominal plar with $k = 2$, $a = 3$, $\zeta = 0.6$, $\omega_n = 4$, $\tau = 0.05$; and the reduce template which differs from (7) in that $a \in [2.5, 3]$.

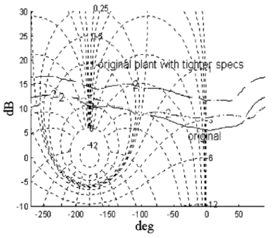


Fig. 10. The Horowitz bounds $\partial B_L(2j)$ for the original template and the original specification ("original"); the original template and tighter specifications ("...tighter specs"); and the reduced template with tighter specifications (unmarked).

Trade-off in feedback design, cont'd

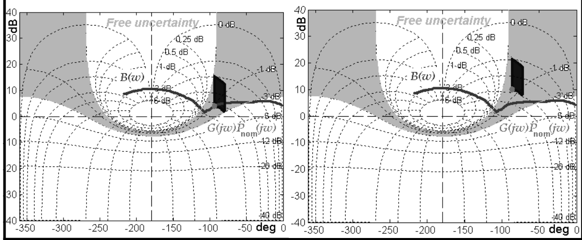
Closed loop specs **Feedback gain**
Trade-off
Plant uncertainty

Adaptation **Fundamental Limitations**
Plant uncertainty **Feedback gain**
Trade-off
Closed loop specs

TECHNION
Israel Institute of Technology

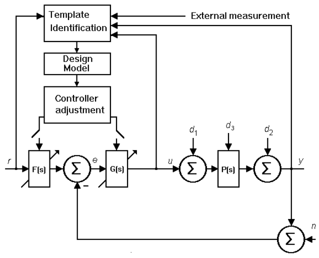
The rôle of adaptation

- Identify the plant templates at constraining frequencies in a helpful way such that the specifications may be satisfied with the least possible feedback gain.



TECHNION
Israel Institute of Technology

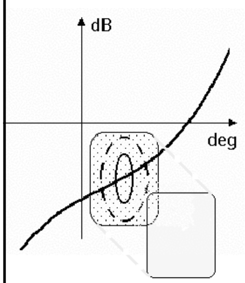
Adaptive Robust Control



- **Switching between robust LTI controllers**
- **Cf. gain scheduled controllers for fighter aircrafts**
  - globally robust backup controller for stability
  - local robust controllers for high performance

TECHNION
Israel Institute of Technology

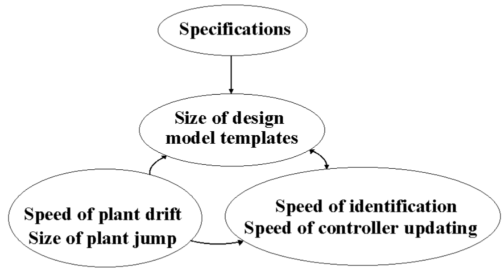
Adaptive Robust Control



- **Identified template —**
- **Probabilistic design template for performance design - - -**
- **Worst case template for stability design only**
- Template extension, due to expected plant jump, or plant drift during the period up to the next possible instant for adaptation (yellow).

TECHNION
Israel Institute of Technology

Adaptive Robust Control, cont'd



TECHNION
Israel Institute of Technology

Adaptive Robust Control, cont'd

- **On-line template identification**
  - "Robust identification"
  - Worst case, set membership, identification
- **On-line (re)design or adjustment of a robust controller**
- Ref. O.Yaniv, P.-O Gutman, L.Neumann, An algorithm for the adaptation of a robust controller to reduced plant uncertainty, Automatica 4 (1990)709 –720.

TECHNION
Israel Institute of Technology

Conclusion

- **Adaptive Control should stick to fidelity.**
- **Robust Control may fiddle around (but not with too fat templates)**