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Robust Control with Classical Methods – QFT

Per-Olof Gutman

- Review of the classical Bode-Nichols control problem
- QFT in the basic Single Input Single Output (SISO) case
- Uncertainty and Fundamental Design Limitations
- QFT for non-minimum phase and computer controlled systems
- QFT for cascaded systems, and for a class of non-linear plants
- QFT for Multi-Input Multi-Output (MIMO) plants
- A comparison between QFT and other robust and adaptive control

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QFT for cascaded SISO systems

- **Canonical 3 d-o-f system**
- **Design objectives**
- **Cascaded SISO – cost of feedback**
- **Digital procedure: outside – in**
  - **Outer loop first**
  - **Free uncertainty**
  - **Inner loop**
- **Example**
- **Exercises**

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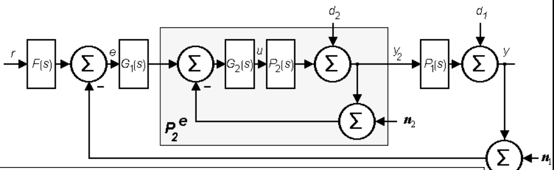
Cascaded 3 d-o-f system with two measurable plant variables



$L_2 = P_2 G_2$
$P_2^e = L_2 / (1 + L_2)$
$L_1 = G_1 P_2^e P_1$
$T = F L_1 / (1 + L_1)$

$\frac{U}{N_1} = \frac{-L_1/(P_1 P_2)}{(1+L_1)} \approx -G_1 G_2, s \to \infty$

$\frac{U}{N_2} = \frac{-L_2/P_2}{(1+L_1)(1+L_2)} \approx -G_2, s \to \infty$

$\frac{Y}{D_1} = \frac{1}{1+L_1}$

$\frac{Y}{D_2} = \frac{1}{(1+L_1)(1+L_2)}$

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Design objectives

- Technical assumption that may be relaxed: P_1 and P_2 have **independent** uncertainties
- Specifications, as in one-section SISO
- Design F, G_1, G_2 s.t. specifications are satisfied
- Use remaining freedom to minimize

$$u_{\text{rms}}^2 = \int_0^\infty \left(\left| \frac{U}{N_1} \right| \Phi_{N_1} + \left| \frac{U}{N_2} \right| \Phi_{N_2} \right) d\omega$$

where Φ_{N_1} and Φ_{N_2} are power spectra of N_1, N_1

- This is often difficult to do exactly, so approximately ...

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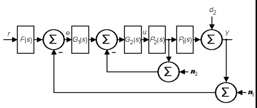
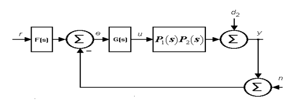
Cascaded SISO system



Cost of feedback:

$|u(s)/n_1(s)| \to |G_1(s) G_2(s)|, \ s \to \infty$

$|u(s)/n_2(s)| \to |G_2(s)|, s \to \infty$



Cf. one-section SISO design:

$|u(s)/n(s)| \to |G(s)|, s \to \infty$

- Trade-off: sensor cost $\leftrightarrow$ cost of feedback
- Sometimes: cascaded solution exists when one-section SISO does not

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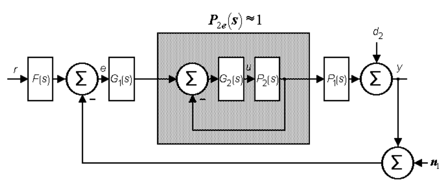
Design procedure Outer loop first

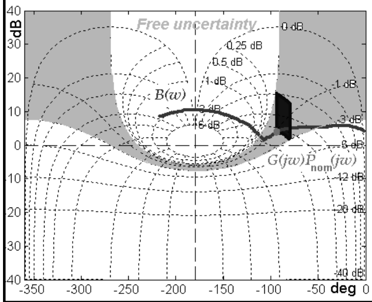


- **Assume temporarily that inner closed loop = 1**
- **Design as in ordinary SISO, but satisfying the Horowitz bounds with some margin, in order to leave some *free uncertainty* for the inner closed loop so that its bandwidth does not have to become too high.**

Free uncertainty

- Given the value set $\{P_i(j\omega)\}$, the computed bound $B(\omega)$, and a feedback compensator $G(j\omega)$, such that the specification is satisfied.
- **Definition**: The **Free Uncertainty** is defined as the maximal value set that includes $\{P_i(j\omega)\}$, that would satisfy the specificaton with the given bound and feedback compensator.

Free uncertainty, cont'd

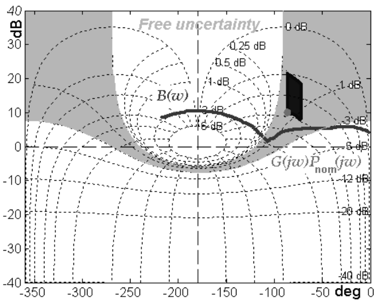


- B(ω) = bound emanating from tolerance specification

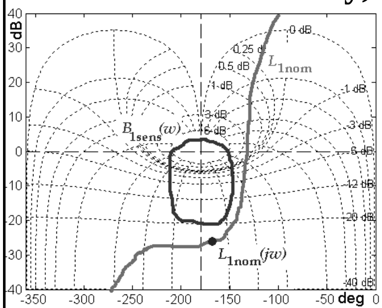
$$\frac{\max\left|S(j\omega)\right|}{\min\left|S(j\omega)\right|} \le \sqrt{2}\,(=3\text{dB})$$

- The Free Uncertainty value set would have satisfied the same spec
- Note that there is no free uncertainty in all directions if the design is "tight" on the bound

Free uncertainty, cont'd



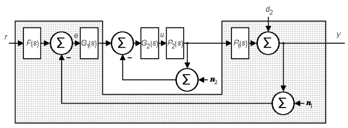
Free uncertainty, cont'd



- When designing the outer nominal loop, $L_{1\text{nom}}$, it is important to leave some margin to the bounds, in order to accomodate for uncertainty in P_2^e

Design procedure: …and then the Inner loop

- **Since $G_1(s)$ is now known, and $P_1(s)$ is known (with uncertainty) the computation of the Horowitz-Sidi bounds is different for the inner loop:**

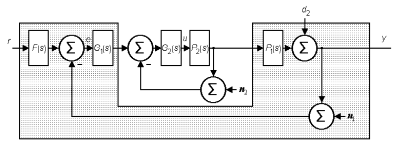


♥ Tolerance bound: compute feasible set for $G_2(s)$.

♥ b and a are the original servo specifications (y/r)

$$\frac{\max\limits_{m,n}\left|\dfrac{G_1(j\omega_k)\left(\dfrac{G_2(j\omega_k)P_{2n}(j\omega_k)}{1+G_2(j\omega_k)P_{2n}(j\omega_k)}\right)P_{1m}(j\omega_k)}{1+G_1(j\omega_k)\left(\dfrac{G_2(j\omega_k)P_{2n}(j\omega_k)}{1+G_2(j\omega_k)P_{2n}(j\omega_k)}\right)P_{1m}(j\omega_k)}\right|}{\min\limits_{m,n}\left|\dfrac{G_1(j\omega_k)\left(\dfrac{G_2(j\omega_k)P_{2n}(j\omega_k)}{1+G_2(j\omega_k)P_{2n}(j\omega_k)}\right)P_{1m}(j\omega_k)}{1+G_1(j\omega_k)\left(\dfrac{G_2(j\omega_k)P_{2n}(j\omega_k)}{1+G_2(j\omega_k)P_{2n}(j\omega_k)}\right)P_{1m}(j\omega_k)}\right|} \le \frac{b(\omega_k)}{a(\omega_k)}$$

Design procedure: the Inner loop



- **Output disturbance rejection bounds, $|y/d_2| \le c$:**

$$\max_{m,n}\left|1+G_1(j\omega_k)\left(\frac{G_2(j\omega_k)P_{2n}(j\omega_k)}{1+G_2(j\omega_k)P_{2n}(j\omega_k)}\right)P_{1m}(j\omega_k)\right|^{-1} \le c(j\omega_k)$$

- **Finally, find F(s) as in ordinary SISO design.**

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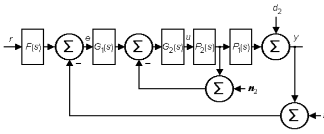
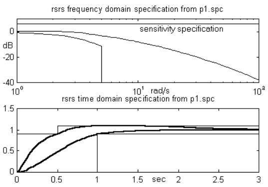
Example



- Servo, and sensitivity (6dB) specifications

$$P_1(s)=\frac{k_1}{(s+1)^2}, k_1\in[1,100]$$

$$P_2(s)=\frac{k_2}{s}, k_2\in[1,100]$$



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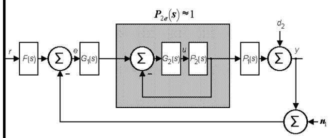
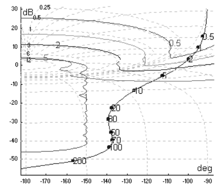
Outer cascaded design of $G_1(s)$





$$G_1(s)=3\frac{(s^2+2s+1)(s/50+1)(s/350+1)}{s(s/10+1)(s/200+1)(s^2/350^2+s/350+1)}$$

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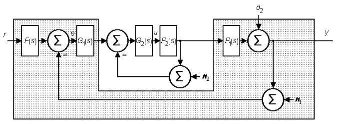
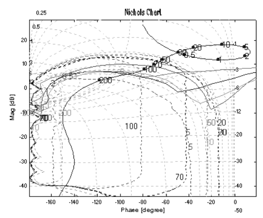
Inner cascaded design of $G_2(s)$





$$G_2(s)=2\frac{(s^2+2s+1)}{(s/5+1)(s/20+1)}\cdot\frac{(s/200+1)(s/800+1)}{(s/3500+1)(s^2/200^2+s/200+1)}$$

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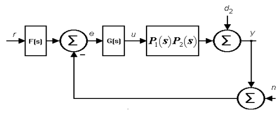
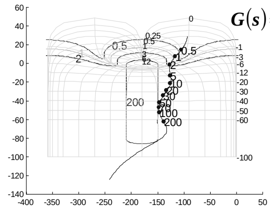
Cf. one-section SISO design





$$G(s)=\frac{2.8(1+s/0.8)(1+s/4)(1+s/200)}{s(1+s/24)(1+s/1800)(1+s/3500)}$$

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Comparison: Cost of feedback

- **Cascaded SISO:**

$$|G_1(500\cdot 2\cdot\pi\cdot j)|=22\mathbf{dB}$$

$$|G_2(500\cdot 2\cdot\pi\cdot j)|=30\mathbf{dB}$$

- **One-section SISO:**

$$|G(500\cdot 2\cdot\pi\cdot j)|=37\mathbf{dB}$$

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Exercises

1. Correct and simplify the design of the example problem. Include the prefilter design. Simulate.
2. Given the plant and specs below. Solve the design problem as an a) cascaded problem, and b) as a one-section SISO problem. Compare!

$$P_1(s)=\frac{K_1}{(s^2+2\varsigma\omega_n s+\omega_n^2)} \qquad P_2(s)=\frac{ABK_2}{(s+a)(s+B)}$$

$$K_1\in[4,1250],\ \varsigma\omega_n\in[-3,5],\ \omega_n\sqrt{1-\varsigma^2}\in[2,10]$$

$$K_2\in[1000,5215],\ A\in[1,3],\ B\in[10,20]$$

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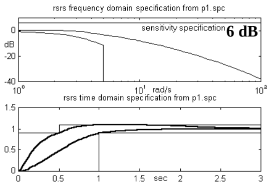
Exercise 2, specifications

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```