KTH Matematik

EXAMINATION IN SF2980 RISK MANAGEMENT, 2017-04-10, 08:00–13:00.

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Allowed technical aids: Everything except computers and communication devices. All books, notes, old exams and similar are allowed. A calculator may be necessary.

Any notation introduced must be explained and defined. Assumptions must be clearly stated. Arguments and computations must be detailed so that they are easy to follow. A table of the Binomial distribution is given at the end of the exam.

GOOD LUCK!

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Problem 1

Let $X = (X_1, \dots, X_d)^T$ be a d-dimensional random (column) vector having an elliptical distribution with location parameter $\mu = (\mu_1, \dots, \mu_d)^T$ and dispersion matrix Σ with full rank. Suppose that the random variable $(X-\mu)^T\Sigma^{-1}(X-\mu)$ has a Gamma(a, b)-distribution. Give a step-by-step instruction of an algorithm for sampling from the joint distribution of X that can easily be implemented in standard software such as R or Matlab. (10 p)

Problem 2

A company is using a model for computing its Value-at-Risk at level 0.05 over a horizon of ten days. The computed Value-at-Risk and the outcomes of the net-worth for 20 independent ten-day periods are given in Table 1 (negative values of the net worth correspond to losses). To backtest the company's model you count the number of times the outcome exceeds the computed Value-at-Risk and the company's model is rejected if there are too many exceedances. Determine if the company's model can be rejected at a confidence level of approximately 98%. You may assume that the interest rate over a ten-day period is 0.

Period	1	2	3	4	5	6	7	8	9	10
VaR	29.2	30.9	30.0	29.2	30.0	29.5	29.4	30.6	29.3	30.4
Net worth	-11.7	-33.9	6.1	17.8	14.2	2.5	1.9	-3.5	-21.5	-47.3
Period	11	12	13	14	15	16	17	18	19	20
VaR	30.1	30.2	31.0	29.6	29.8	31.0	28.2	29.9	29.5	29.5
Net worth	8.0	-18.8	-7.7	18.1	22.7	0.7	-35.0	-31.5	-11.2	32.0

Table 1: Value-at-Risk and outcome of net worth in Problem 2.

Problem 3

Historical observations of the daily claim amount (in million SEK) for a type of insurance are collected. A quantile-quantile plot of the logarithm of the daily claim amount (y-axis) versus the quantiles of a standard Exponential distribution (x-axis) is given in Figure 1. Estimate the probability that the aggregated claim amount over 30 days exceeds 60. Assumptions must be clearly stated.

Problem 4

In the Solvency II regulatory framework the so-called standard formula for computing the solvency capital requirement (SCR) is given by

$$\text{Basic SCR} = \sqrt{\sum_{i=1}^{d}\sum_{j=1}^{d} \text{Corr}_{i,j} \times \text{SCR}_i \times \text{SCR}_j}, \tag{1}$$

where Basic SCR is the capital requirement for the insurance company with $d = 5$ modules (Market risk, Default risk, Life risk, Health risk and Non-life risk), $\text{SCR}_i, i = 1, \dots, d$ is the capital requirement for each module and $\text{Corr}_{i,j}$ are predefined correlation parameters.

Derive the standard formula (1) from the assumption that the joint distribution of net-worth for each module $(X_1, \dots, X_d)$ has a centered elliptical distribution with correlation matrix C such that $C_{ij} = \text{Corr}_{i,j}$, where $\text{SCR}_i = \text{VaR}_{0.005}(X_i)$ and $\text{Basic SCR} = \text{VaR}_{0.005}(X_1 + \cdots + X_d)$. Here centered means that the vector of location parameters of the elliptical distribution is 0.

Problem 5

Let (U_1, U_2, U_3) be a three-dimensional random vector having $\text{U}(0, 1)$ marginal distributions. Let $C_{12}(u_1, u_2)$, $C_{13}(u_1, u_3)$ and $C_{23}(u_2, u_3)$ denote the copulas of the pairs (U_1, U_2), (U_1, U_3) and (U_2, U_3), respectively. Is it possible that (U_1, U_2, U_3) has a joint distribution such that C_{12}, C_{12} and C_{12} are all counter-monotonic? Give a complete motivation for your answer by either specifying an appropriate joint distribution $C(u_1, u_2, u_3)$ or by demonstrating that such a construction is impossible.

(10 p)

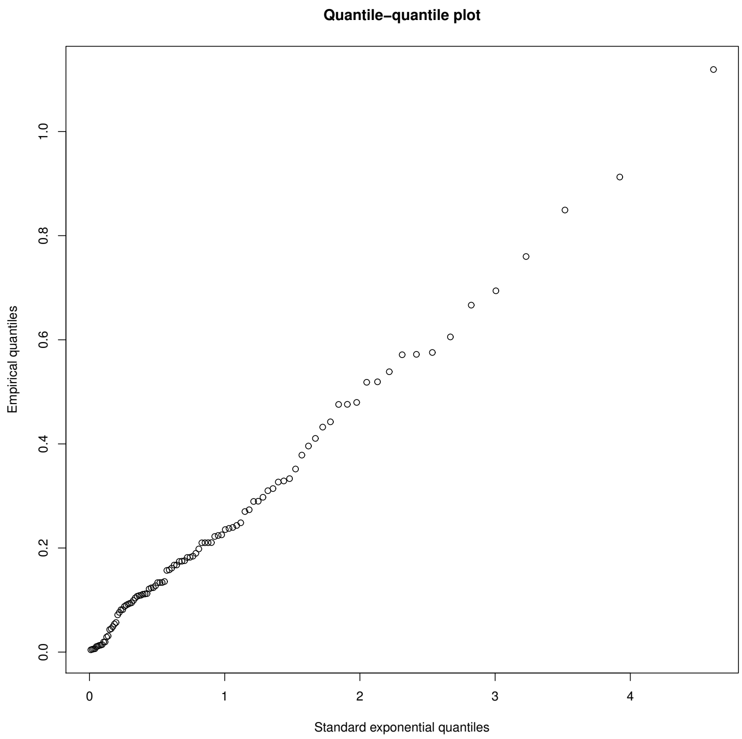

Figure 1: This figure relates to Problem 3. It shows a quantile-quantile plot over the logarithm of the daily claim amount (y-axis) versus the quantiles of a standard Exponential distribution (x-axis)

TABLE OF THE BINOMIAL C.D.F.

n	x	p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
20	0		0.8179	0.6676	0.5438	0.4420	0.3585	0.2901	0.2342	0.1887	0.1516
	1		0.9831	0.9401	0.8802	0.8103	0.7358	0.6605	0.5869	0.5169	0.4516
	2		0.9990	0.9929	0.9790	0.9561	0.9245	0.8850	0.8390	0.7879	0.7334
	3		1.0000	0.9994	0.9973	0.9926	0.9841	0.9710	0.9529	0.9294	0.9007
	4		1.0000	1.0000	0.9997	0.9990	0.9974	0.9944	0.9893	0.9817	0.9710
	5		1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9981	0.9962	0.9932
	6		1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9987
	7		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
	8		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n	x	p	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
20	0		0.1216	0.0388	0.0115	0.0032	0.0008	0.0001	0.0000	0.0000	0.0000
	1		0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001	0.0000
	2		0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002
	3		0.8670	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049	0.0013
	4		0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189	0.0059
	5		0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553	0.0207
	6		0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299	0.0577
	7		0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520	0.1316
	8		0.9999	0.9987	0.9900	0.9591	0.8867	0.7624	0.5956	0.4143	0.2517
	9		1.0000	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914	0.4119
	10		1.0000	1.0000	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507	0.5881
	11		1.0000	1.0000	0.9999	0.9991	0.9949	0.9804	0.9435	0.8692	0.7483
	12		1.0000	1.0000	1.0000	0.9998	0.9987	0.9940	0.9790	0.9420	0.8684
	13		1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9935	0.9786	0.9423
	14		1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9793
	15		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9941
	16		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987
	17		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
	18		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000