EXAMINATION IN SF2980 RISK MANAGEMENT, 2014-01-14, 14:00–19:00.

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Allowed technical aids: Everything except computers and communication devices. All books, notes, and similar are allowed. A calculator is necessary.

Any notation introduced must be explained and defined. Assumptions must be clearly stated. Arguments and computations must be detailed so that they are easy to follow.

GOOD LUCK!

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Problem 1

$X = V_1 - V_0 = 100(e^Z - 1)$, where Z is the daily logreturn. With $L = -X = 100(1 - e^Z)$ we have

$$\mathrm{VaR}_{0.01}(X) = F_L^{-1}(0.99) = -F_X(0.01) = 100(1 - e^{F_Z^{-1}(0.01)}).$$

Since $\Phi^{-1}(0.01) = -\Phi^{-1}(0.99) = -2.33$ we see from Figure 1 that $F_Z^{-1}(0.01) \approx -0.06$. Therefore

$$\mathrm{VaR}_{0.01}(X) = 100(1 - e^{-0.06}) = 5.82.$$

Problem 2

(a) First we have that $P(I_1 = 1, I_2 = 1) = P(Y_1 \leq F^{-1}(p), Y_2 \leq F^{-1}(p)) = C(p,p)$. Since C is the Clayton copula we get $P(I_1 = 1, I_2 = 1) = (2p^{-\theta} - 1)^{-1/\theta}$, and consequently

$$\mathrm{Cor}(I_1, I_2) = \frac{E[I_1 I_2] - E[I_1]E[I_2]}{\sqrt{\mathrm{Var}(I_1)\mathrm{Var}(I_2)}} = \frac{P(I_1 = 1, I_2 = 1) - p^2}{p(1-p)} = \frac{(2p^{-\theta} - 1)^{-1/\theta} - p^2}{p(1-p)}.$$

(b) It follows from the calculations in Section 9.5.1 that

$$a = \frac{1-c}{c}p, \quad b = \frac{1-c}{c}(1-p),$$

where $c = \mathrm{Cor}(I_1, I_2)$ is the answer in (a).

Problem 3

It follows from the formulation that

$$V_1 = 10^6(e^{Y_1} - e^{Y_2}),$$

where we can write

$$Y_1 = \mu_1 + \sigma_1 Z_1,$$
$$Y_2 = \mu_2 + \sigma_2(\rho Z_1 + \sqrt{1-\rho^2} Z_2).$$

In polar coordinates we can write

$$Z_1 = r\sin\theta, \quad Z_2 = r\sin\theta,$$

which implies that $\Delta V_1 = V_1 - V_0$ can be written as a function of θ as

$$\Delta V_1(\theta) = 10^6(\exp\{\mu_1 + r\cos\theta\} - \exp\{\mu_2 + \sigma_2(\rho r\cos\theta + \sqrt{1-\rho^2} r\sin\theta\}).$$

Linearizing gives $e^x \approx 1 + x$ and hence

$$\Delta V_1^{\text{lin}}(\theta) = 10^6(\mu_1 + r\cos\theta - \mu_2 - \sigma_2(\rho r\cos\theta + \sqrt{1-\rho^2} r\sin\theta).$$

Minimizing with respect to θ gives (differentiating and put equal to zero)

$$0 = 10^6 r(-\sigma_1\sin\theta - \sigma_2(-\rho\sin\theta + \sqrt{1-\rho^2}\cos\theta).$$

Solving for θ gives

$$\tan\theta = \frac{\sigma_2\sqrt{1-\rho^2}}{\rho\sigma_2 - \sigma_1},$$

and therefore

$$\theta = \arctan\Big(\frac{\sigma_2\sqrt{1-\rho^2}}{\rho\sigma_2 - \sigma_1}\Big) + k\pi, \quad k = \dots - 1, 0, 1, \dots.$$

An investigation of the derivative shows that the minimum in the interval $[0, 2\pi]$ is at

$$\theta^* = \arctan\Big(\frac{\sigma_2\sqrt{1-\rho^2}}{\rho\sigma_2 - \sigma_1}\Big) + \pi.$$

With $r^2 = 25$ and the numerical values of the parameters inserted the corresponding scenario for Y_1 and Y_2 is

$$Y_1 = -0.017, \quad Y_2 = -0.0045,$$

which is the answer to (a). The corresponding (linearized) loss is

$$\Delta V_1^{\text{lin}}(\theta^*) = -12229.$$

This is the answer to (b).

Problem 4

(a) Using linearization we have

$$\Delta V^{\text{lin}} = 10^6(Y_1 - Y_2).$$

With $\mathbf{w} = 10^6(1, -1)^T$ we get

$$\Delta V^{\text{lin}} = \mathbf{w}^T\mathbf{Y} =^d \mathbf{w}^T\boldsymbol{\mu} + \sqrt{\mathbf{w}^T A A^T \mathbf{w}} Z_1,$$

where Z_1 has a standard t_3 distribution. It follows that

$$\text{VaR}_p(\Delta V^{\text{lin}}) = -\mathbf{w}^T\boldsymbol{\mu} + \sqrt{\mathbf{w}^T A A^T \mathbf{w}}\, t_3^{-1}(p).$$

That is,

$$p = 1 - t_3\Big(\frac{10^4 + \mathbf{w}^T\boldsymbol{\mu}}{\sqrt{\mathbf{w}^T A A^T \mathbf{w}}}\Big).$$

With numerical values inserted we get $p = 1 - t_3(4.06) = 0.013$
(b) Replacing 10^4 by 12229 we get $p = 1 - t_3(5) = 0.0077$.

Problem 5

Using the subexponential approximation as in Example 8.18 it follows that

$$F_{X_D}^{-1}(u) \approx F_Y^{-1}\Big(1 - \frac{1-u}{n^{1-\alpha}}\Big),$$

where $F_Y^{-1}(v) = (1-v)^{-1/\alpha}$. This implies that

$$\begin{aligned}
\text{ES}_p(X_D) &\approx \frac{1}{p}\int_0^p F_Y^{-1}\Big(1 - \frac{u}{n^{1-\alpha}}\Big)du \\
&= \frac{1}{p}\int_0^p \Big(\frac{u}{n^{1-\alpha}}\Big)^{-1/\alpha} du \\
&= \frac{\alpha}{\alpha - 1} p^{-1/\alpha} n^{1/\alpha - 1}.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\text{ES}_p(X_1) &\approx \frac{1}{p}\int_0^p F_Y^{-1}(1-u)du \\
&= \frac{1}{p}\int_0^p u^{-1/\alpha} du \\
&= \frac{\alpha}{\alpha - 1} p^{-1/\alpha}.
\end{aligned}$$

We conclude that $\delta_p(n) \approx n^{1/\alpha - 1}$ for small p. (10 p)