SF2942: ASSIGNMENT 1

Due on September 23, 10:00 am.

General information

This is the first of two voluntary homework assignments in the course. To warrant any bonus points for the final exam a solution must be well-written and handed in on paper (unless an exception has been made beforehand). Late submissions will not be considered. You may work in groups of at most three students. Collaborations between students / groups are encouraged, but each group must hand in their own set of solutions. Any group may be asked to explain their solutions and failure to do so will result in a reduction in the bonus points awarded to that group. For additional guidelines please see the course syllabus.

Introduction

In the course we have only discussed European derivatives. The notion of an *American* derivative is similar, with the difference being that the owner of a contract may choose to exercise at any time before maturity. For now we let $c_t^A(T,K)$ and $p_t^A(T,K)$ denote the prices at time t of an American call and put, respectively, on some underlying asset with spot price S_t, with maturity T and strike K. We will not discuss American derivatives in great detail, but the following results, which can be seen as no-arbitrage conditions, may prove useful in the problems:

- The price of an American call option, $c_t^A(T,K)$, is non-decreasing in the time to maturity: If $T_1 < T_2$, then $c_t^A(T_1,K) \leq c_t^A(T_2,K)$, for $t \in [0,T_1]$.
- $c_t^A(T,K)$ is a convex function of the strike.
- For a pure investment asset, e.g., a non-dividend paying stock, the price of an American call option is the same as a that of a European call option.
- A version of the put-call parity, with inequality, holds for American option prices:

$$c_t^A(T,K) - p_t^A(T,K) \leq S_t - d_{t,T}K, \ \forall t \in [0,T],$$

  where $d_{t,T}$ is the discount factor from time t to time T (known at time t).

For more information on American options, as well as proofs of the claims above, see the note by Filip Lindskog available on the webpage.

Problem 1

Consider 11 interest rate swaps with the common nominal amount $L = 1,000,000$ dollars and maturities $1,2,3,5,7,10,12,15,20,25,30$ years. Each swap pays yearly fixed-rate payments and floating-rate-payments every six months. The first floating-rate-payment is in six months. The k-year swap pays the fixed amount $c_k L$ yearly, the first payment is in one year and the last payment is in k years. The c_ks are given (in %) in Table 1.

Table 1. Swap rates in %.

c_1	c_2	c_3	c_5	c_7	c_{10}	c_{12}	c_{15}	c_{20}	c_{25}	c_{30}
0.54	0.66	0.87	1.44	1.94	2.43	2.65	2.86	3.02	3.1	3.15

From the swap rates, determine the discount factors d_t for $t \in \{1, 2, \ldots, 10\}$ years and the corresponding zero-rates r_t.

Problem 2

Consider the American IBM stock option prices (in \$) at closing on April 26 in Table 2. The IBM share price at that time was \$205.58. A dividend of \$0.85 per share is paid on June 9 to stock holders of record May 10. The current zero rates for all maturities less than three months is assumed to be 1% per year. The prices in Table 2 are last prices and not necessarily prices at which you can trade. Assume, however, that you can trade at these prices and take both long and short positions in the options, the stock, and in zero-coupon bonds with arbitrary maturities. For the computation of discount factors, assume that a year consists of 365 days.

Determine whether or not there are any inconsistencies, that is arbitrage opportunities, in the option prices in Table 2. Explain what methods you use, and if you find any inconsistencies where (i.e., what prices) they are and what principle is violated.

Table 2. Prices of American stock options at closing on April 26.

Strike	Last	Vol	Open Int	Last	Vol	Open Int
Call, exp. at close May 18				Put, exp. at close May 18		
190.00	15.95	82	1440	0.16	507	4430
195.00	10.9	282	1321	0.37	539	4328
200.00	6.2	1300	4186	1.05	1481	4761
205.00	2.7	4161	14152	2.72	1297	3200
210.00	0.82	4189	8032	5.9	1188	2225
215.00	0.22	359	5585	10.35	37	780
Call, exp. at close June 15				Put, exp. at close June 15		
190.00	16	53	219	0.78	278	592
195.00	11.35	104	571	1.36	901	622
200.00	7.5	758	1428	2.4	621	589
205.00	4.32	961	1442	4.4	994	270
210.00	2.11	1157	3784	6.94	119	145
215.00	0.9	567	1001	10.95	23	103

Problem 3

At the start of September 7 the spot price for a Share of Swedbank A was 195.20 (SEK). Prices for American call options on one share of Swedbank A, with maturity on September 30, 2016, are given in Table 3. In reality these prices were computed as mid-prices, but

Table 3. Prices for call options on Swedbank A for different strikes K on September 7.

C_0	11.200	7.125	3.800	1.750	0.750
K	185	190	195	200	205

we assume for now that you can trade at them. The stock pays no dividends between September 7 and September 30. Complete the following tasks:

- Compute and plot the implied volatilities obtained from the prices in Table 3.
- Fit a volatility smile for strikes $K \in [160, 230]$. Plot this together with the implied probabilities and provide the form of your fitted volatility smile, including coefficients.
- Compute and plot the implied forward density obtained for your fitted volatility smile. Compare (in a plot) to those given by Black's model when you use the mean of the computed implied volatilities as the volatility-parameter.

You can assume that the current zero-rate for the time period in question is 0.5% per year and that there are 252 trading days per year.