LECTURE 1
CH.4.1 FROM AVNER FRIEDMAN
FOUNDATIONS OF MORDERN ANALYSIS

Definitions: Linear vector space. Linear independent elements. Infinite dimension. Basis in Lenear vector space. Coordinates. Normed linear space. Metric space. Fréchet space.

Theorem.
A normed lenear space is a metric linear space, s.t. $\rho(x, y) = \|x - y\|$.

Definition. Complete vector space. Banach space. Equivalent norms.

Home exercises.
1. Show that if $x = \{\xi_n\}_{n=1}^{\infty}$, $y = \{\eta_n\}_{n=1}^{\infty}$, then

$$\rho(x, y) = \sum_{n+1}^{\infty} \frac{1}{2^n} \frac{|\xi_n - \eta_n|}{1 + |\xi_n - \eta_n|}$$

is a metric space, but not a Fréchet space.

2. *(ex. 4.1.1 from AFr)*
Let X be a normed liner space. Show that $B = \{x \in X : \|x\| < 1\}$ is convex.

3. *(ex. 4.1.2 from AFr)*
Let X be a linear space and $K, F \subset X$ are convex. Show that $K + F$ is convex.

4. We define Lorentz classes $L_{p,s}(\Omega)$, $1 < p < \infty$, $0 < s \leq \infty$, $\Omega \subset \mathbb{R}^n$ as follows:
Let

$$\mu_f(t) = \text{meas}\{x \in \Omega : |f(x)| > t\}$$

$$f \in L_{p,s}(\Omega) \qquad \iff \qquad t\mu_f^{1/p}(t) \in L_s(\mathbb{R}_+, t^{-1}dt).$$

Quasi-norm in $L_{p,s}(\Omega)$ is given by

$$\|f\|_{p,s} = \left(\int_0^{\infty} t^{s-1} \mu_f^{s/p}(t)\, dt\right)^{1/s}.$$

Show that $L_{p,p}(\Omega) = L^p(\Omega)$.