Homework 1 Due date October 17th

Due date October 17th, send it by e-mail at wojtek@math.kth.se or put it in my mailbox in the math department

Problem 1. (10 points). Let R be a commutative ring and $\mathbf{Z}$ be the group of integers. Show that there is an isomorphism of R-algebras $R[\mathbf{Z}]$ and $R[X, X^{-1}]$.

Problem 2. Let R be a commutative ring and G a group.

(1) (5 points). Show that there is a ring homomorphism $\pi : R[G] \to R$ such that for any $g \in G$, $\pi(g) = 1$.
(2) (10 points). Let I be the kernel of π. Show that if a set S generates the group G, then the elements $s - 1$ for $s \in S$ generate I as a left ideal.
(3) (15 points). Show that if S is a subset of G such that $s - 1$ (for $s \in S$) generate I as a left ideal, then S generates the group G.

Problem 3. (10 points). Show that if R is a commutative ring such that all the ideals are principal, then for any multiplicative set $S \in R$, all the ideals in $R[S^{-1}]$ are also principal.

Problem 4. (15 points). Show that the group of invertible elements in the ring $\mathbf{Z}/2^n$ is isomorphic to $\mathbf{Z}/2 \times \mathbf{Z}/2^{n-2}$.

Problem 5. (15 points). Let $p > 2$ be a prime number. Show that the group of invertible elements in the ring $\mathbf{Z}/p^n$ is cyclic. What is its order?