

Morita Theorem

Theorem 1. (Morita) *Let R and S be rings. The categories of R -modules and S -modules are equivalent if and only if there is a S - Mod - R module M such that:*

- a. *M is finitely generated projective S -module;*
- b. *the map $R \rightarrow \text{hom}_S(M, M)$ is an isomorphism of rings;*
- c. *any S module is isomorphic to $M \otimes_R N$ for some R -module N .*

In the above theorem, the conditions a and b say that the functor $M \otimes_R - : R\text{-Mod} \rightarrow S\text{-Mod}$ is a bijection on the sets of morphisms. The third condition c says that all the S -modules are isomorphic to modules in the image of this functor.