Problem 1. Let $0 \to M_0 \to M_1 \to M_2 \to 0$ and $0 \to N_0 \to N_1 \to N_2 \to 0$ be exact sequences of R-modules. Suppose that we have a commutative diagram of the form:

Show that if any two of f_0 , f_1 , f_2 are isomorphisms, then so is the third.

Problem 2. Consider the following commutative diagram of R-modules with exact rows:

Show that if f_1 , f_2 , f_4 , and f_5 are isomorphisms, then f_3 is also and isomorphism.

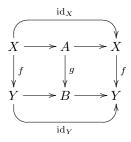
Problem 3. Give examples of exact sequences of abelian groups:

$$0 \to A_0 \to A_1 \to A_2 \to 0$$
 , $0 \to B_0 \to B_1 \to B_2 \to 0$

such that:

- a. $A_0 \simeq B_0$, $A_1 \simeq B_1$, and $A_2 \not\simeq B_2$.
- b. $A_0 \simeq B_0, A_1 \not\simeq B_1, \text{ and } A_2 \simeq B_2.$
- c. $A_0 \not\simeq B_0$, $A_1 \simeq B_1$, and $A_2 \simeq B_2$.

Problem 4. We say that a morphism $f: X \to Y$ is a retract of $g: A \to B$ if there is a commutative diagram of the form:



Show that if g is an isomorphism, epimorphism, monomorphism of R-modules then so is any of its retract.

Problem 5. Describe the following as abelian groups:

$$\hom_{\mathbf{Z}}(\mathbf{Z},\mathbf{Z}/n) \quad \hom_{\mathbf{Z}}(\mathbf{Z}/m,\mathbf{Z}/n) \quad \hom_{\mathbf{Z}}(\mathbf{Z}/n,\mathbf{Z}) \quad \hom_{\mathbf{Z}}(\mathbf{Q},\mathbf{Z}) \quad \hom_{\mathbf{Z}}(\mathbf{Q},\mathbf{Q})$$

Problem 6. Let $0 \to M_0 \xrightarrow{f} M_1 \xrightarrow{g} M_2 \to 0$ be an exact sequence of R-modules. Show that the following statements are equivalent:

- a. There exists $s: M_2 \to M_1$ such that $gs = \mathrm{id}_{M_2}$.
- b. There exists $t: M_2 \to M_1$ such that $tf = id_{M_0}$.

c. The following is an exact sequence for any R-module N:

$$0 \to \hom_R(N, M_0) \xrightarrow{f_*} \hom_R(N, M_1) \xrightarrow{g_*} \hom_R(N, M_2) \to 0$$

d. The following is an exact sequence for any R-module N:

$$0 \to \hom_R(M_2,N) \xrightarrow{g^*} \hom_R(M_1,N) \xrightarrow{f_*} \hom_R(M_0,N) \to 0$$

e. There is an R-module homomorphism $t:M_2\to M_1$ such that the homomorphism $f\oplus t:M_0\oplus M_2\to M_1,\ (a,b)\mapsto f(a)+t(b)$, is an isomorphism.