Homework 3 Due date May 23rd

Due date May 23rd, send it by e-mail at wojtek@math.kth.se or put it in my mailbox in the math department

Solve either problems 1 and 3 or problems 2 and 3

Problem 1. Show that $\pi_1(S^2, x) = 0$.

Problem 2.

(a) Let $x, y \in S^2$ be such that $x \neq -y$. The arc between x and y is a map $\tau : [0,1] \to S^2$ given by the formula:

$$\tau(t) = \frac{ty + (1-t)x}{|ty + (1-t)x|}$$

Show that for any finite set of arcs $\tau_1, \tau_2, \ldots, \tau_n$ in S^2

$$\tau_1([01]) \cup \tau_2([01]) \cup \cdots \cup \tau_n([01]) \neq S^2$$

(S^2 is not a union of images of **finitely** many arcs).

(b) Let $\omega : [0,1] \to S^2$ be a map such that for any $t \in [0,1]$, $|\omega(t) - \omega(0)| < 1/3$. Show that ω is homotopic relative $\{0,1\}$ to the the arc $\tau : [0,1] \to S^2$ where:

$$\tau(t) = \frac{t\omega(1) + (1-t)\omega(0)}{|t\omega(1) + (1-t)\omega(0)|}$$

(c) Let $\omega : [0,1] \to S^2$ be a map. Show that there is n such that, for any $0 \leq k \leq n$ and any $t \in [k/n, (k+1)/n] \subset [0,1]$, $|\omega(t) - \omega(k/n)| < 1/3$.

(d) Show that any map $\omega : I \to S^2$ is homotopic relative $\{0,1\}$ to a path of the form $\tau_1 * \tau_2 * \cdots * \tau_n$ for some arcs τ_i.

- Show that $\pi_1(S^2, x) = 0$.

Problem 3. Calculate $\pi_1(\mathbf{P}^2, *)$, where $\mathbf{P}^2$ is the projective space