Homework 2 Due date May 9th

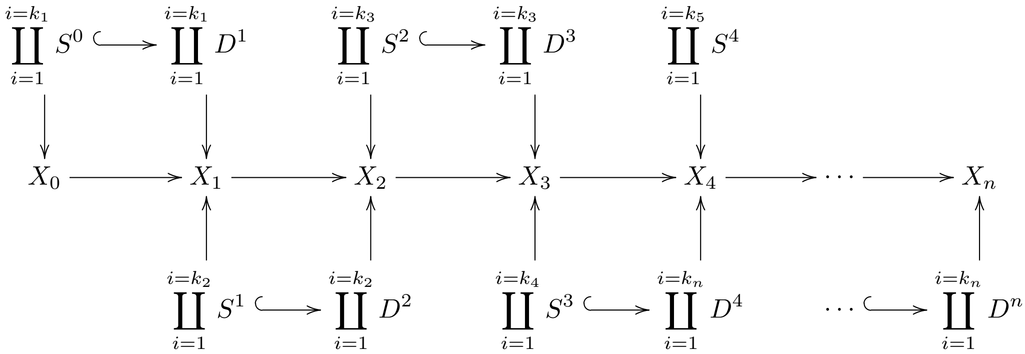
Due date May 9th, send it by e-mail at wojtek@math.kth.se or put it in my mailbox in the math department

Problem 1. Prove that the subspace:

$$\{(x^2 - y^2, xy, xz, yz) \in \mathbf{R}^4 \mid x, y, z \in \mathbf{R} \text{ and } x^2 + y^2 + z^2 = 1\} \subset \mathbf{R}^4$$

is homeomorphic to the projective space $\mathbf{P}^2$.

Recall that a CW complex is a diagram of the form:



where all the squares are push-outs and X_0 is a space with a discreet topology. Elements of X_0 are called vertices of the CW-complex. The number:

$$|X_0| - k_1 + k_2 - k_3 + \cdots + (-1)^n k_n$$

is called the Euler characteristic of the CW complex. A CW decomposition of a topological space X is a CW complex for which X is homeomorphic to X_n.

Problem 2. Calculate the Euler characteristic of the standard CW decomposition of the projective space $\mathbf{P}^n$.

Problem 3. Describe two CW decompositions of S^n. One with only one vertex and one with 2 vertices. Calculate the Euler characteristic of these CW complexes.

Problem 4. Describe two CW decompositions of $S^1 \times S^1$. One with only one vertex and one with 3 vertices. Calculate the Euler characteristic of these CW complexes.