Challenging problems, February 7

Do not get discourage if you can not solve these problems. Try to have fun thinking about them. Write why you find them interesting or not interesting.

Problem 3. Let G be a finite group.

(1) Show that if G is cyclic, then for any $n > 0$, the set $\{g \in G \mid g^n = e\}$ has at most n elements.
(2) Assume that, for any $n > 0$, the set $\{g \in G \mid g^n = e\}$ has at most n elements. Is G abelian? Is G cyclic?