

Challenging problems, January 31

Do not get discouraged if you can not solve these problems. Try to have fun thinking about them. Write why you find them interesting or not interesting.

Problem 1. Let G be a set with an associative binary operation and $e \in G$ an element satisfying the following conditions:

- (1) $eg = g$ for any $g \in G$.
- (2) For any g there is h such that $hg = e$.

Show that G with this binary operation is a group.

Problem 2. Let G be a set with an associative binary operation and $e \in G$ an element satisfying the following conditions:

- (1) $eg = g$ for any $g \in G$.
- (2) For any g there is h such that $gh = e$.

Is G with this binary operation a group? If not find an example which is not a group.

Assume that p is a prime number and G has p -elements. How many non isomorphic such binary operations are on G which are not groups?