FÖREL NR3: Sammansatta - inversa fkner

Def 1. f, g Säges vara sammsatt om $h(x) = f[g(x)]$ en fkn tecknas $h = f\circ g$

EX1 $f(x) = \sqrt{x}$, $D(f): x \geq 0$

$g(x) = x+1$, $D(g): x \in \mathbb{R}$

$\cdot (f\circ g)(x) = f[g(x)] = \{ y = g(x) \} = f(y)$

$= \sqrt{y} = [y = x+1] = \sqrt{x+1}$

$D(f\circ g) = x+1 \geq 0, \ x \geq -1$

$\cdot (g\circ f)(x) = g[f(x)] = g(v) = [v = f(x) = \sqrt{x}]$

$= v+1 = \sqrt{x} + 1$

$(f\circ g)(x) = \sqrt{x+1}$

$(g\circ f)(x) = \sqrt{x} + 1$

$g\circ f \neq f\circ g$

En bild

$$x \to \boxed{f} \xrightarrow{y = f(x)} \boxed{g} \to g(y) = g[f(x)]$$

$$x \to \boxed{g} \xrightarrow{y = g(x)} \boxed{f} \to f(y) = f[g(x)]$$

$$g[f(x)] \neq f[g(x)]$$

om $g[f(x)] = f[g(x)]$ alla x

Så säges att f och g är invers av varandra

Om inversen

Def 2 $f : D(f) \longrightarrow V(f)$

$x \longrightarrow y = f(x)$

f säges vara **omvändbar** (injektiv, 1-1)

om för $x_1, x_2 \in D(f)$

och $f(x_1) = f(x_2) \Longrightarrow x_1 = x_2$

dvs f omvändbar $\Longleftrightarrow$ varje $y \in V(f)$

är bilden av endast $\boxed{\text{ett}}$ $x \in D(f)$

Geom. f omvändbar $\Longleftrightarrow$

varje rätlinje $y =$ konstant

skär grafen till f högst

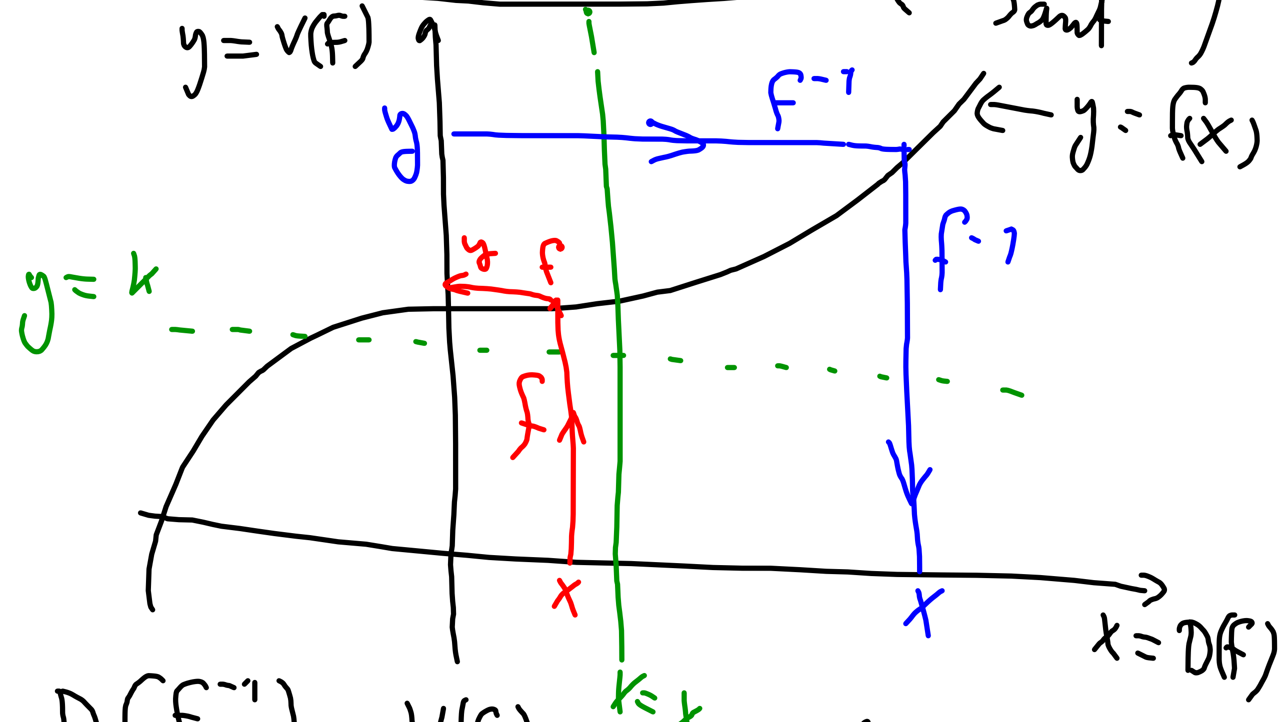
en gång!

Vi tecknar inversen till f med f^{-1}

$$f: D(f) \longrightarrow V(f)$$

$$f^{-1}: V(f) \longrightarrow D(f)$$

obs! $\boxed{f^{-1}(x) \neq \frac{1}{f(x)}}$ $\left(2^{-1} = \frac{1}{2} \text{ sant}\right)$



$$D(f^{-1}) = V(f) = y\text{-axeln}$$

$$V(f^{-1}) = D(f) = x\text{-axeln}$$

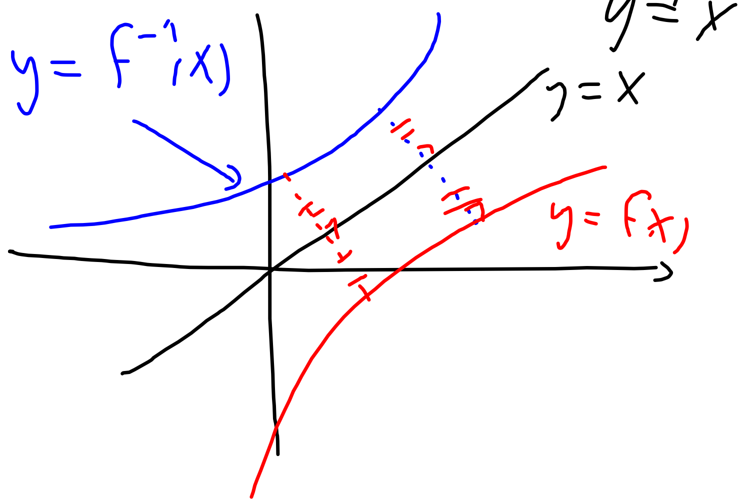
$$x \to \boxed{f} \xrightarrow{y=f(x)} \boxed{f^{-1}} \to x = f^{-1}(y) = f^{-1}(f(x)) = (f^{-1}\circ f)(x)$$

$$y \to \boxed{f^{-1}} \xrightarrow{x=f^{-1}(y)} \boxed{f} \to y = f(x) = f[f^{-1}(y)] = (f\circ f^{-1})(y)$$

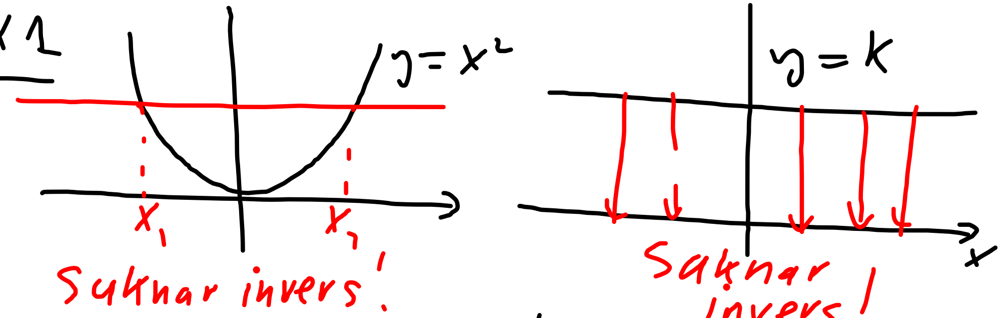
$$(f^{-1}\circ f)(x) = x \quad (1)$$

$$(f\circ f^{-1})(y) = y \quad (2)$$

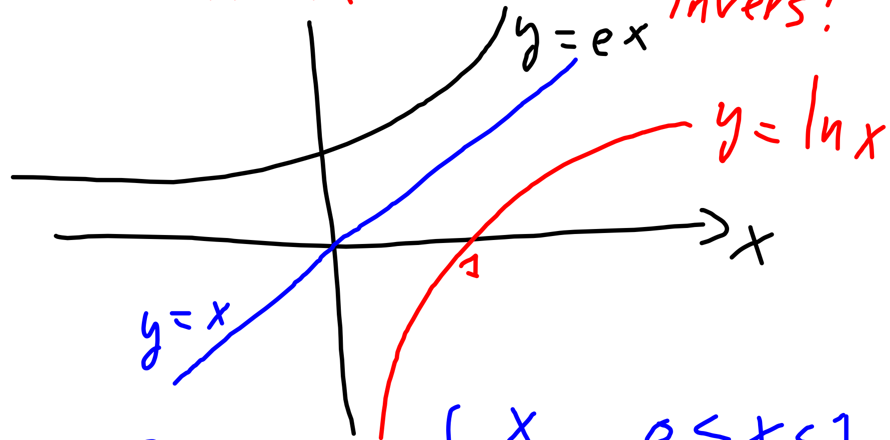
(1) och (2) säger att kurvan $y = f^{-1}(x)$ är spegelbild av kurvan $y = f(x)$ map $y = x$



EX 1

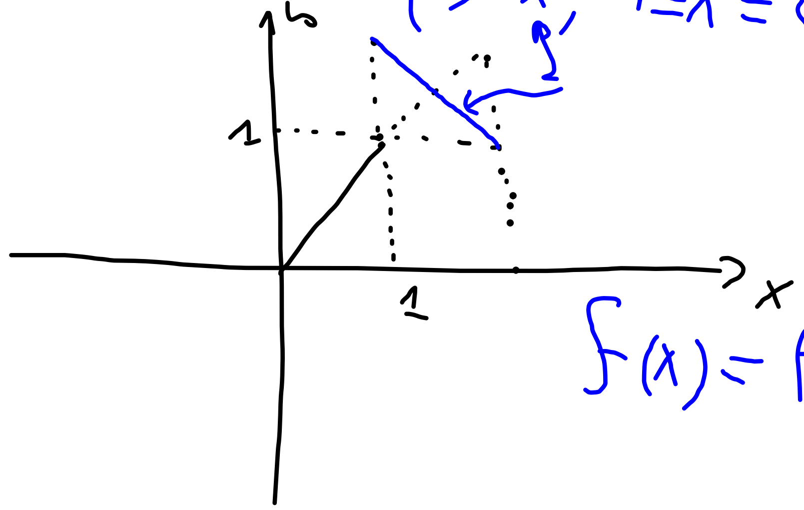


EX 2



EX 3

$$f(x) = \begin{cases} x, & 0 \le x < 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$$



$f(x) = f^{-1}(x)$

<u>Lär in EX</u> Visa att $f(x) = \frac{1+x}{x}$, $x>0$
har en invers och ange denna på
formen $y = f^{-1}(x)$.

<u>Lösning</u> $f(x) = \frac{1+x}{x} = 1 + \frac{1}{x}$, $x>0$

a) f har en invers om $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
för $x_1 > 0$, $x_2 > 0$

$$1 + \frac{1}{x_1} = 1 + \frac{1}{x_2} \Longrightarrow x_1 = x_2$$

$\therefore f^{-1}$ finns

b) Hur finner man inversen på formen
$$y = f^{-1}(x)$$

$y = 1 + \frac{1}{x}$ vi löser $x = g(y)$

Vi får

$$y = \frac{x+1}{x} \Rightarrow yx = x + 1 \Longrightarrow$$

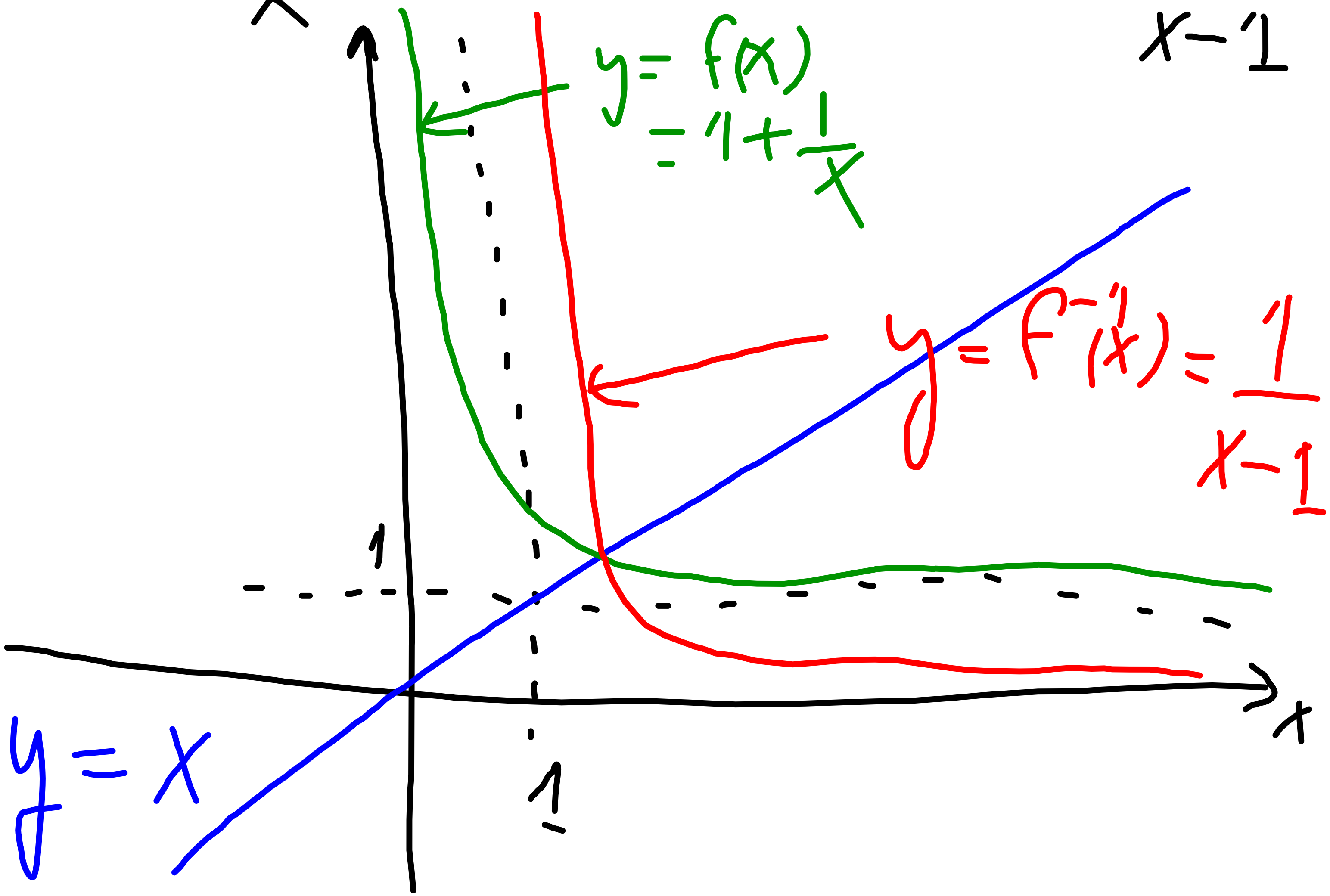
$$yx - x = 1 \Longleftrightarrow x(y-1) = 1$$

$$\Longrightarrow \boxed{x = \frac{1}{y-1} = g(y)} \quad (*)$$

Vill man ha $y = f^{-1}(x)$
byt x mot y i $(*)$

$$y = \frac{1}{x-1} = f^{-1}(x)$$

$y = 1 + \frac{1}{x}$ har inveren $f^{-1}(x) = \frac{1}{x-1}$



Arcusfunktionerna är inverser till de trigonometriska grundfunktionerna

1. $y = \sin(x)$, $-\frac{\pi}{2} \leq x \leq \pi/2$, $-1 \leq y \leq 1$

inversen är $f^{-1}(x) = \arcsin(x)$ (Fig sid 116)

$$\boxed{y = \sin(x) \iff x = \arcsin(y)}$$

$y = \sin(x) = \sin(\arcsin(y))$, $-1 \leq y \leq 1$

$x = \arcsin(y) = \arcsin(\sin(x))$, $-\frac{\pi}{2} \leq x \leq \pi/2$

2. $y = \cos x \qquad 0 \le x \le \pi, \quad -1 \le y \le 1$

inversen är $f^{-1}(x) = \arccos(x)$ (fig sid 118)

$$\boxed{y = \cos x \iff x = \arccos(y)}$$

$$y = \cos(x) = \cos(\arccos(y)), \quad -1 \le y \le 1$$

$$x = \arccos(y) = \arccos(\cos x), \quad 0 \le x \le \pi$$

3. $y = \tan x, \ -\frac{\pi}{2} < x < \pi/2 \ ; -\infty < y < \infty$

inversen är $f^{-1}(x) = \arctan(x)$

$$\boxed{y = \tan(x) \iff x = \arctan(y)}$$

$y = \tan(x) = \tan(\arctan(y)), \ -\infty < y < \infty$

$x = \arctan(y) = \arctan(\tan(x)), \ -\frac{\pi}{2} < x < \pi/2$

4. $y = \cot(x), \ 0 < x < \pi, \quad y \in \mathbb{R}$

Inversen $\boxed{f^{-1}(x) = \operatorname{arccot}(x) = \frac{\pi}{2} - \arctan x}$

$$\boxed{y = \cot(x) \iff x = \operatorname{arccot}(y)}$$

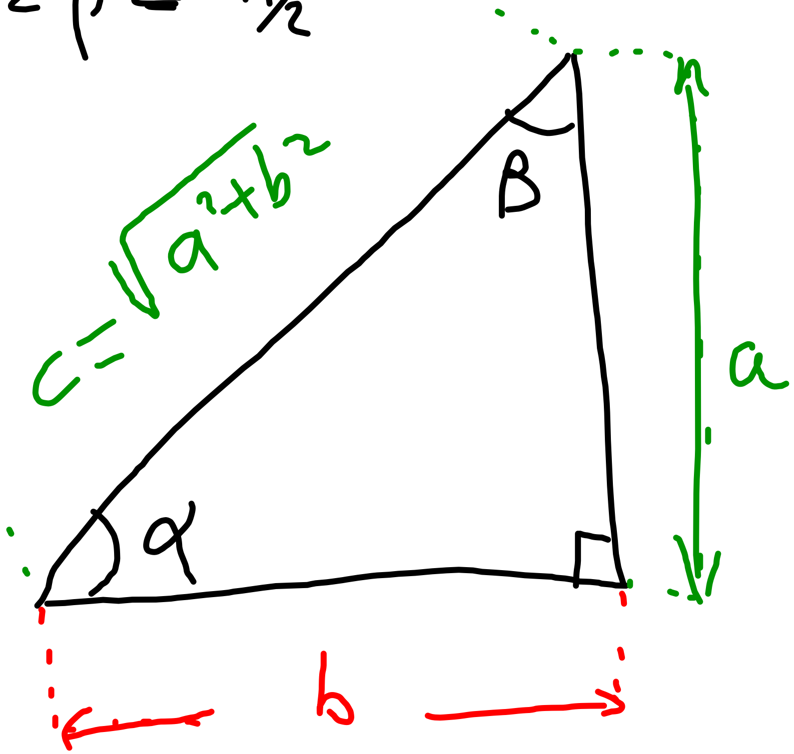
$y = \cot(\operatorname{arccot}(y)) \quad -\infty < y < \infty$

$x = \operatorname{arccot}(y) = \operatorname{arccot}(\cot(x))$

$0 < x < \pi$

ATT FÖRSTÅ Arcus funktionerna

$0<\alpha<\pi/2 \quad , \quad 0<\beta \le \pi/2$



$$\sin\alpha = \frac{a}{c}$$

$$\Updownarrow$$

$$\alpha = \arcsin\left(\frac{a}{c}\right)$$

$$\cos\alpha = \frac{b}{c}$$

$$\Updownarrow$$

$$\alpha = \arccos\left(\frac{b}{c}\right)$$

$$\tan\alpha = \frac{a}{b} \Leftrightarrow \alpha = \arctan\left(\frac{a}{b}\right)$$

$$\cot\alpha = \frac{1}{\tan\alpha} = \frac{b}{a} \Leftrightarrow \alpha = \operatorname{arccot}\left(\frac{b}{a}\right)$$

EX Beräkna $\arcsin(\frac{1}{2}) + \arcsin(\frac{\sqrt{3}}{2})$

Lösning

Sätt $\alpha = \arcsin(\frac{1}{2}) \Leftrightarrow \sin\alpha = \frac{1}{2}$

$\Rightarrow \alpha = \pi/6$

$\beta = \arcsin(\frac{\sqrt{3}}{2}) \Leftrightarrow \sin(\beta) = \frac{\sqrt{3}}{2}$.

$\Rightarrow \beta = \pi/3$

SVAR $\alpha + \beta = \arcsin(1/2) + \arcsin(\frac{\sqrt{3}}{2}) =$

$\pi/6 + \pi/3 = \pi/2$

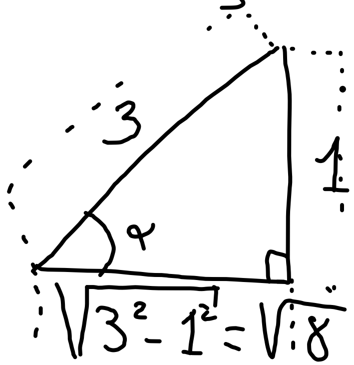
EX Beräkna $\cos(\arcsin(1/3))$

Lösning sätt

$\alpha = \arcsin(\frac{1}{3}) \Leftrightarrow \sin\alpha = 1/3$

Vi söker $\cos(\alpha)$

$\cos(\alpha) = \frac{\sqrt{8}}{3} < 1$



SVAR $\cos(\arcsin(1/3))$

$= \sqrt{8}/3$

LärinEX Beräkna

$$\sin\left(\arcsin\left(\frac{7}{8}\right)+\arccos\left(\frac{1}{4}\right)\right) \quad (*)$$

Lösning

Sätt $\alpha = \arcsin\left(\frac{7}{8}\right) \iff \sin\alpha = \frac{7}{8}$

$\beta = \arccos\left(\frac{1}{4}\right) \iff \cos(\beta) = 1/4$

$(*) \iff \sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$= \frac{7}{8}\cdot\frac{1}{4} + \sin\beta\cos\alpha$

m $\sin^2 A + \cos^2 A = 1$

$$\sin\beta = \sqrt{1-\cos^2\beta} = \sqrt{1-\frac{1}{4^2}} = \sqrt{1-\frac{1}{16}} = \sqrt{\frac{16-1}{16}} = \frac{\sqrt{15}}{4}$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\left(\frac{7}{8}\right)^2} = \sqrt{1-\frac{49}{64}} = \frac{\sqrt{64-49}}{8} = \frac{\sqrt{15}}{8}$$

$$\sin(\alpha+\beta) = \frac{7}{32} + \frac{\sqrt{15}}{8}\cdot\frac{\sqrt{15}}{4} = \frac{7+15}{32} = \frac{22}{32} = \frac{11}{16}$$

SVAR $\sin\left(\arcsin\left(\frac{7}{8}\right)+\arccos\left(\frac{1}{4}\right)\right) = \frac{11}{16}$