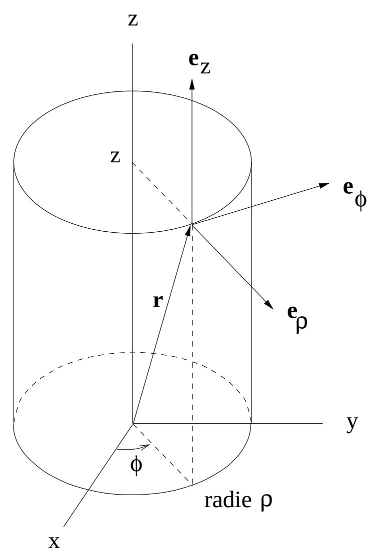
Kroklinjiga koordinater

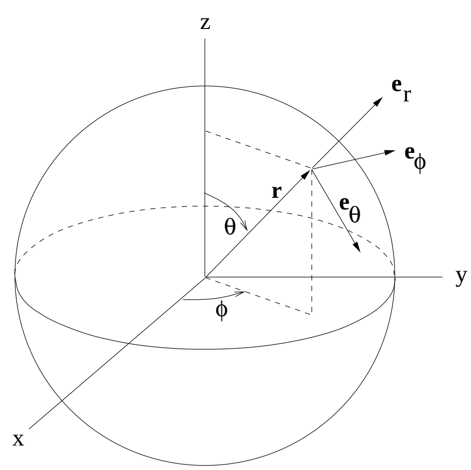
- Byt från det Kartesiska systemet x, y, z med enhetsvektorer $\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_y, \hat{\mathbf{e}}_z$ till nya variabler u_1, u_2, u_3 med enhetsvektorer $\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3$ (som antas vara ett ON högersystem)
- Basvektorer $\hat{\mathbf{e}}_i = \frac{1}{h_i}\frac{\partial \mathbf{r}}{\partial u_i}$ Skalfaktorer $h_i = \left|\frac{\partial \mathbf{r}}{\partial u_i}\right|$
- Cylindriska koordinater

$x = \rho\cos\phi$ $\hat{\mathbf{e}}_\rho = (\cos\phi, \sin\phi, 0)$ $h_\rho = 1$
$y = \rho\sin\phi$ $\hat{\mathbf{e}}_\phi = (-\sin\phi, \cos\phi, 0)$ $h_\phi = \rho$
$z = z$ $\hat{\mathbf{e}}_z = (0, 0, 1)$ $h_z = 1$
$h_\rho h_\phi h_z = \rho$

- Sfäriska koordinater

$x = r\sin\theta\cos\phi$ $\hat{\mathbf{e}}_r = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ $h_r = 1$
$y = r\sin\theta\sin\phi$ $\hat{\mathbf{e}}_\theta = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta)$ $h_\theta = r$
$z = r\cos\theta$ $\hat{\mathbf{e}}_\phi = (-\sin\phi, \cos\phi, 0)$ $h_\phi = r\sin\theta$
$h_r h_\theta h_\phi = r^2\sin\theta$





SKALFAKTORERNA INGÅR I INTEGRALER OCH DERIVATOR:

- $\int_C \mathbf{A}\cdot d\mathbf{r} = \int_{C'} \sum_{i=1}^3 A_i\, h_i\, du_i$
- $\iint_S \mathbf{A}\cdot d\mathbf{S} = \iint_{S'} A_3\, h_1\, h_2\, du_1\, du_2$ (om $\hat{\mathbf{n}}$ antas $= \hat{\mathbf{e}}_3$)
- $\iiint_V \phi\, dV = \iiint_{V'} \phi\, h_1\, h_2\, h_3\, du_1\, du_2\, du_3, \quad h_1\, h_2\, h_3 = \left|\frac{\partial(x,y,z)}{\partial(u_1,u_2,u_3)}\right| =$ Jacobianen
- $\text{grad}\,\phi = \sum_{i=1}^3 \frac{1}{h_i}\frac{\partial\phi}{\partial u_i}\hat{\mathbf{e}}_i$ OBS: $\neq \frac{\partial\phi}{\partial u_1}\hat{\mathbf{e}}_1 + \frac{\partial\phi}{\partial u_2}\hat{\mathbf{e}}_2 + \frac{\partial\phi}{\partial u_3}\hat{\mathbf{e}}_3$
- $\text{div}\,\mathbf{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial A_1h_2h_3}{\partial u_1} + \frac{\partial h_1A_2h_3}{\partial u_2} + \frac{\partial h_1h_2A_3}{\partial u_3}\right]$ OBS: $\neq \frac{\partial A_1}{\partial u_1} + \frac{\partial A_1}{\partial u_2} + \frac{\partial A_1}{\partial u_3}$
- $\text{rot}\,\mathbf{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{\mathbf{e}}_1 & h_2\hat{\mathbf{e}}_2 & h_3\hat{\mathbf{e}}_3 \\ \partial_1 & \partial_2 & \partial_3 \\ h_1A_1 & h_2A_2 & h_3A_3 \end{vmatrix}$ OBS: $\neq \begin{vmatrix} \hat{\mathbf{e}}_1 & \hat{\mathbf{e}}_2 & \hat{\mathbf{e}}_3 \\ \partial_1 & \partial_2 & \partial_3 \\ A_1 & A_2 & A_3 \end{vmatrix}$
- $\Delta\phi = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\frac{h_2h_3}{h_1}\frac{\partial\phi}{\partial u_1} + \text{c.p.}\right]$ OBS: $\neq \frac{\partial^2\phi}{\partial u_1^2} + \frac{\partial^2\phi}{\partial u_2^2} + \frac{\partial^2\phi}{\partial u_3^2}$