Problem73# 2.2 Solution

The object is to show that, assuming $\sum (\xi_i/a_i)^2 = 1$, the equation $\sum[(\xi_i x_i)/(a_i)^2] = 1$ has only the solution $x_i = \xi_i$, $i = 1, 2, \ldots, n$.

Now, by Cauchy's inequality
$$1 = \left\{\sum[(\xi_i/a_i)(x_i/a_i)]\right\}^2 \leq \sum[(\xi_i/a_i)^2] \sum[(x_i/a_i)]^2$$
and we have strict inequality unless $x_i = t\xi_i$ for all i, and some scalar t.

Since x is to lie on E we must have $t = 1$ or -1. Only the choice $t = 1$ gives x on the required hyperplane.

HSS