Problem73# 2.1 Solution

In case $n = 2$ we get:

$(x_1^2 + x_2^2)(y_1^2 + y_2^2) - (x_1y_1 + x_2y_2)^2 =$

$x_1^2y_1^2 + x_1^2y_2^2 + x_2^2y_1^2 + x_2^2y_2^2 - x_1^2y_1^2 - x_2^2y_2^2 - 2x_1x_2y_1y_2 =$

$x_1^2y_2^2 + x_2^2y_1^2 - 2x_1x_2y_1y_2 = (x_1y_2 - x_2y_1)^2$

In general we have (summation from $j, k = 1$ up to n):

$$A = (\sum x_jy_j)^2 = \sum x_j^2y_j^2 + \sum_{j \neq k} x_jy_jx_ky_k.$$
$$B = (\sum x_j^2)(\sum y_j^2) = \sum x_j^2y_j^2 + \sum_{j<k} x_j^2y_k^2 + \sum_{j>k} x_j^2y_k^2.$$
$$B - A = \sum_{j<k} x_j^2y_k^2 + \sum_{j<k} x_k^2y_j^2 - 2\sum_{j<k} x_jx_ky_jy_k =$$
$$\sum_{j<k}(x_jy_k - x_ky_j)^2.$$

Hence $\quad (\sum x_j^2)(\sum y_j^2) - (\sum x_jy_j)^2 = \sum_{j<k}(x_jy_k - x_ky_j)^2.$

This relation is known as Lagrange's identity and of course implies Cauchy's inequality:

$$(\sum x_jy_j)^2 \leq (\sum x_j^2)(\sum y_j^2).$$

GJ