SF1611 Introductory course in mathematics I. 1.5 cr
Solutions to the sample exam. Duration: 60 minutes. No aids allowed

The problems are worth 1 credit each and you are only required to provide answers, not complete derivations. In order to pass, you must get at least 5 credits.

Name:..Pers.no...................................Program.............

Result:

1	2	3	4	5	6	7	8	Σ	**Grade**

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1. Write in words how the following statement is pronounced.

$$\forall\delta > 0 \, \exists\varepsilon > 0, \; |x| < \varepsilon \; \Rightarrow \; |\sin x| < \delta$$

Answer: *For any delta greater than zero there is an epsilon greater than zero such that, if the absolute value of x is less than epsilon, then the absolute value of sine x is less than delta.*

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2. Use mathematical symbols to define the set of all real numbers whose distance from 7 on the number line is strictly greater than 2.
Answer: $\{x \in \mathbb{R} \; : \; |x - 7| > 2\}$ or $\mathbb{R} \setminus [5, 9]$

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3. The third-degree polynomial $x^3 - 4x^2 - 7x + 10$ has a zero at $x = 1$. Find the remaining zeros.
Answer: $x = -2$ and $x = 5$

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4. Find all positive solutions to the equation $x - 2 = \sqrt{x}$.
Answer: $x = 4$

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5. Find an integer n such that $|\frac{n}{5} - e| < \pi - 3$.
Answer: $n = 13$ or $n = 14$

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6. Simplify $e^{2\ln 2}$ as much as possible.
   Answer: 4

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7. Find all real solutions to the equation $\cos 2x = 1/2$.
   Answer: $x = \pm\frac{\pi}{6} + n\pi$ where $n \in \mathbb{Z}$.

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8. The Fibonacci number sequence $f_0, f_1, f_2, \ldots$ begins with 0 and 1 and after that every entry is the sum of the two previous ones:

$$0, 1, 1, 2, 3, 5, 8, 13, \ldots$$

Fill in the gap in the following proof that $f_n < 2^n$ for any natural number n.

> Induction over n. For $n = 0$ and $n = 1$ the statement is true since $f_0 = 0 < 1 = 2^0$ and $f_1 = 1 < 2 = 2^1$. Under the assumption that the statement holds for $n-1$ and $n-2$ we want to show that it holds also for n.
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> By definition $f_n = f_{n-1} + f_{n-2}$, and by the induction assumption we have
>
> $$f_{n-1} < 2^{n-1} \text{ and } f_{n-2} < 2^{n-2}.$$
>
> Thus
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> $$f_n = f_{n-1} + f_{n-2} < 2^{n-1} + 2^{n-2} < 2^{n-1} + 2^{n-1} = 2^n.$$
