Problem session November 6, SF2736, fall 12.

Please prepare!

1. Is the following information sufficient to find the relation $\mathcal{R}$:
   1. The relation $\mathcal{R}$ is an equivalence relation on $\mathcal{M} = \{1, 2, 3, 4, 5, 6\}$.
   2. $\{(1, 2), (2, 3), (2, 4), (5, 6)\} \subseteq \mathcal{R}$.
   3. $(2, 6) \notin \mathcal{R}$.

2. Let $\mathcal{M}$ denote the set $\{1, 4, 5, 8, 11, 12, 13, 17\}$. For any two elements $a, b \in \mathcal{M}$ define $a\mathcal{R}b$ if 4 divides $b - a$. Show that $\mathcal{R}$ is an equivalence relation on $\mathcal{M}$ and describe the equivalence classes.

3. Which is the mistake in the following proof for that a relation $\mathcal{R}$ which is symmetric and transitive must be reflexive: If $a\mathcal{R}b$ then by symmetry $b\mathcal{R}a$ and hence by transitivity $a\mathcal{R}b$ and $b\mathcal{R}a$ will imply that $a\mathcal{R}a$

4. Assume that $f : A \to B$ and $g : B \to A$ are such that
$$(g \circ f)(x) = x \qquad \text{for all} \qquad x \in A\ .$$
Will this imply that f and g, respectively, are either injective, surjective or bijective?

5. (a) Let $f : A \to B$, $g : B \to C$ and $h : C \to D$. Is it always true that for every $x \in A$
$$((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)\ .$$
   (b) Find two functions $f : A \to A$ and $g : A \to A$ such that
$$f \circ g \neq g \circ f\ .$$
   (c) Show that if $f : A \to B$ and $g : B \to C$ are bijective functions then the function $g \circ f : A \to C$ will be a bijective function.

6. (a) Show that a union of any finite family of countable infinite sets is a countable infinite set.
   (b) Can the union of a countable infinite family of countable sets be countable infinite.

7. Let S be a set of five positive integers the maximum of which is at most 9 show that the sums of the elements in all the nonempty subsets of S cannot all be distinct.

8. Show that to any sequence a_1, a_2, ..., a_{p+1} of $p+1$ positive integers there will always exist a subsequence
$$a_{i_1},\ a_{i_2}, \ldots,\ a_{i_t}\ ,$$
such that
$$a_{i_1} + a_{i_2} + \ldots + a_{i_t} \equiv 0 \pmod{p}\ .$$

9. Are there any non-prime numbers n, with $n > 1$, such that
$$m^{n-1} \equiv 1 (\text{mod } n)$$
for every integer m such that n does not divide m.