Problem session December 6, SF2736, fall 12.

Please prepare!

1. Show that there is no graph with the following sequence of degrees 2, 3, 3, 3, 3, 4, 5 of its vertices.

2. Are the following two graphs isomorphic?

a	b	c	d	e	f
b	a	a	a	b	c
c	c	b	e	d	d
d	e	f	f	f	e

respectively

1	2	3	4	5	6
2	1	2	3	2	1
4	3	4	5	4	3
6	5	6	1	6	5

3. An acyclic graph has 124 vertices and 98 edges. Find the number of components.

4. Show that a graph with n vertices, such that the sum of the degrees of any to vertices is at least equal to $n-1$, must be connected.

5. Find the maximum number of vertices of in a graph with 28 edges if the degree (valency) of every vertex is at least 3.

6. Find orderings of the vertices of the cube for which the greedy algorithm requires 2, 3 and 4 colors respectively, for a coloring where adjacent vertices have distinct colors.

7. Find a complete matching in the bipartite graph on the set of vertices $X = \{a_1, a_2, \ldots, a_5\}$ and $Y = \{b_1, b_2, b_3, \ldots, b_5\}$ and edges $\{(a_1, b_2), (a_1, b_3), (a_2, b_1), (a_2, b_2)$, $(a_2, b_4), (a_3, b_3), (a_3, b_5), (a_4, b_1), (a_4, b_2), (a_4, b_4), (a_5, b_3)\}$.

8. A network and a flow is defined as above

(x,y)	(s,a)	(s,b)	(s,c)	(a,b)	(a,d)	(b,c)	(b,d)	(b,e)	(c,e)	(d,t)	(e,t)
$c(x,y)$	5	6	8	4	10	2	3	11	6	9	4
$f(x,y)$	5	6	0	0	5	1	2	3	1	7	4

   (a) What is the value of f?

   (b) Find an f-augmenting path and compute the value of the augmented flow.

   (c) Find a cut with capacity 12.

9. Suppose that every boy in a school has a list of k girls he can date and suppose that every girl appears on k such lists. Show that every boy can find a girl to date.

10. Show that for every bipartite graph with n vertices it is true that $e \leq (\frac{n}{2})^2$.

11. Find the number of regular 4-valent graphs with seven vertices.

12. Show that if a graph G is not connected then the complement $\bar{G}$ of the graph must be connected.

13. Show that if $\bar{G}$ is the complement of the graph G then $\chi(G)\chi(\bar{G}) \geq n$ where n is the number of vertices of G.