Problem session October 30, SF2736, fall 12.

Please prepare!!

0. Find the inverse of the element 76 in the ring Z_{221}.

1. Solve

   (a) $13x + 18 = 13$ in the ring Z_{64}.

   (b)
   $$\begin{cases} 7x & + & 2y & = & 5 \\ 10x & + & 7y & = & 3 \end{cases}$$
   in the ring Z_{13}.

2. Find all elements b of Z_{17} such that the equation
   $$x^2 + 3x + b = 0 \ ,$$
   has a solution.

3. Find
   $$545^{112} \pmod{23} \qquad \text{and} \qquad 545^{112} \pmod{24} \ .$$

4. Find non-negative integers $n > 2$ and m such that n is not a prime number but n divides $m^{n-1} - 1$.

5. Show that if p is a prime number then
   $$(p-1)! \equiv -1 \pmod{p} \ .$$

6. For any two integers a and b show that the two integers $a/\gcd(a,b)$ and $b/\gcd(a,b)$ are relatively prime.

7. Find all integers n and m such that $314n + 218m = 12$ with $0 \le n \le 20$.

8. Find the number of solutions to the equation $x^2 = 1$ in the ring Z_{990}.

9. Solve the equation $(x-5)(x+3) = 0$ in the ring Z_{56}.

10. (a) Assume that m, n and k are integers with $n \ge 2$ and $m \ge 2$, and assume that $n^2 = km^2$. Prove that k is a square of some integer.

    (b) Assume that $\gcd(x,y) = 1$. Prove that if $xy = z^2$ for some integer z, then $x = n^2$ and $y = m^2$ for some integers n and m.

11. Show that if $2^n - 1$ is a prime number then n must be a prime number. Will the same statement be true if we substitute 2 by any integer $a \ge 2$.

12. (a) Find the number of solutions to an equation
    $$a_1x_1 + a_2x_2 + \ldots + a_nx_n = b \ ,$$
    in a ring Z_p, where p is a prime number.

    (b) Give a more general result from which your answer to the previous problem follows.