Problem session November 14, SF2736, fall 12.

Please prepare!

1. In how many ways can we choose a committee in a class consisting of 11 girls and 12 boys if
   (a) the committee shall consist of 4 girls and 4 boys.
   (b) the committee shall consist of 4 girls and 4 boys, but if the boy A is chosen to the committee then the girl B cannot attend.
2. Find the number of ways we can form words of length 7 using the letters in the word DISKRET if none of the words RET, SIK or DIS may appear as subwords in the word.
3. Show that
$$\binom{m+n}{r} = \binom{m}{0}\binom{n}{r} + \binom{m}{1}\binom{n}{r-1} + \ldots + \binom{n}{r}\binom{n}{0}$$
4. Assume that $r > k$. Prove that
$$\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}.$$
5. Find the coefficient of x^{12} in the polynomial $(4+3x^2)^{10}$.
6. Find a formula for $S(n,2)$.
7. Show that if $\gcd(n,m) = 1$ then $\phi(nm) = \phi(n)\phi(m)$
8. Find the number of positive integers d that divides the integer 129600.
9. In how many ways can fifteen children in a class be placed into three (unlabeled) rows.
10. Find the number of surjective maps f from the set $\{1,2,3,\ldots,10\}$ to $\{1,2,3,\ldots,6\}$ such that the elements $f(1)$, $f(2)$ and $f(3)$ are distinct.
11. Find the number of ways to divide the set $\{1,2,3,4,5,6\}$ into three non-empty subsets in such a way that the elements 1 and 2 will belong to distinct subsets.
12. If eight distinct dices are rolled, what is the probability that all six numbers appear?
13. Find the number of solutions to the Diophantine equation
$$x_1 + x_2 + x_3 + x_4 \leq 15$$
if we require that $0 \leq x_1 \leq 4$, $-2 \leq x_2 \leq 3$ and $2 \leq x_3 \leq 8$.
14. In how many ways can five girls and five boys be divided into three groups in such a way that each group will contain at least one boy and one girl.