Matematiska Institutionen
KTH

Homework number 5 to SF2736, fall 2012.

Please, deliver this homework at latest on Monday, December 3.
    The homework must be delivered individually, and, in general, just hand-written notes are accepted. You are free to discuss the problems below with your class mates, but you are not allowed to copy the solution of another student.

1. (0.2p) Use the technique with generating functions to find explicit expressions for the numbers a_n, $n = 0, 1, 2, ...$, if this sequence of numbers satisfies
$$a_n = 3a_{n-2} + 2a_{n-3}, \qquad n = 3, 4, 5...$$
and $a_0 = 2$, $a_1 = 0$ and $a_2 = 7$.

2. (0.2p) How many distinct necklaces with 12 beads can you form by using just red, green and blue beads.

3. We consider binary words $\bar{c} = a_1 a_2 \ldots a_{11}$ of length 11 and where $a_i \in Z_2$. Let $\mathbf{H}$ denote the matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Let C be the set of words $\bar{c}$ such that $\mathbf{H}\bar{c}^T = \bar{0}^T$, if we count modulo 2.

(a) (0.2p) The code C is an e-error-correcting code. Find e.

(b) (0.2p) Can the word 01101001010 be corrected. If "yes" correct it. If "no" explain why.

(c) (0.2p) Find the number of words in Z_2^{11} that cannot be corrected by this code C.