Matematiska Institutionen
KTH

Homework number 3 to SF2736, fall 2012.

Please, deliver this homework at latest on Tuesday, November 20.

The homework must be delivered individually, and, in general, just handwritten notes are accepted. You are free to discuss the problems below with your class mates, but you are not allowed to copy the solution of another student.

1. (0.2p) In how many ways can the elements in set $\{1, 2, 3, \ldots, 15\}$ be distributed into the three sets A, B and C such that
$$|A \cap B| = |A \cap C| = |B \cap C| = 1.$$

2. (0.2p) In how many ways can the set $\{1, 2, 3, \ldots, 10\}$ be divided into five non-empty subsets such that the elements 1, 2 and 3 belong to distinct subsets.

3. (a) (0.1p) For how many 4-tuples (n_1, n_2, n_3, n_4) are the multinomial coefficients
$$\binom{12}{n_1, n_2, n_3, n_4}$$
defined, as the number of ways to partition a set of 12 elements into four labeled subsets, each with n_1, n_2, n_3 and n_4 elements, respectively.

   (b) (0.2p) Find
$$\sum 3^{n_1}(-6)^{n_2}9^{n_3}(-4)^{n_4}\binom{12}{n_1, n_2, n_3, n_4}$$
where the sum is taken over all 4-tuples (n_1, n_2, n_3, n_4) for which the multinomial coefficients above are defined. The answer must be given as an integer.

4. (0.3p) A class consists of eight girls and nine boys. In how many ways can the children be distributed in the three class rooms D2, E2 and Q2 in such a way that each of the class rooms contains at least two boys and two girls.