Matematiska Institutionen
KTH

Homework number 2 to SF2736, fall 2012.

Please, deliver this homework at latest on Wednesday, November 14.

1. (0.1p) Let $\mathcal{R}$ be an equivalence relation on a set A. Assume that $|A| = 45$ and assume that $\mathcal{R}$ induces a partition of A into five equivalence classes of equal size. Find $|\mathcal{R}|$.

2. (0.2p) Let $\mathcal{R}$ be a relation on a set A which is both transitive and symmetric. Define
$$C_a = \{x \in A \mid a\mathcal{R}x\}.$$
Is the following true
$$C_a \neq C_b \qquad \Longrightarrow \qquad C_a \cap C_b = \emptyset.$$

3. Let $A = \{1, 2, \ldots, 9\}$ and let $\mathcal{R}$ be the following relation on A:
$$\mathcal{R} = \{(1,1), (2,3), (3,5), (7,6), (7,7), (8,9)\}.$$
   (a) (0.1p) Find the smallest equivalence relation that contains $\mathcal{R}$.
   (b) (0.2p) Find the number of equivalence relations that contain $\mathcal{R}$.

4. (0.1p) Find and describe a bijective map that maps the set of real numbers in the open interval $(3, 7)$ onto the interval $(2, 3)$.

5. (0.3p) Assume that A is a given countable infinite set and let B be the set of all real numbers x that are solutions to some polynomial equation
$$a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n = 0,$$
where $a_i \in A$, for $i = 0, 1, \ldots, n$. Is the set B countable infinite?