Matematiska Institutionen
KTH

Homework number 1 to SF2736, fall 2012.

Please, deliver this homework at latest on Tuesday, November 6.

The homework must be delivered individually, and, in general, just hand written notes are accepted. You are free to discuss the problems below with your class mates, but you are not allowed to copy the solution of another student.

1. (0.2p) Find all solutions to the equation $12x = 8$ in the ring Z_{128}.

2. (a) (0.1p) Prove that the sum of any three consecutive integers is divisible by 3. (For example the sum of 7, 6 and 5 is 18, which is divisible by 3.)

   (b) (0.2p) Generalize this fact, i.e., find and prove a more general result from which the result above follows.

3. (a) (0.1p) Give a definition of the greatest common divisor of a set of n non-zero integers $a_1, a_2, \ldots, a_n$, denoted by $\gcd(a_1, a_2, \ldots, a_n)$.

   (b) (0.2p) Use your definition above to prove that, for any three non-zero integers a_1, a_2 and a_3,

   $$\gcd(a_1, a_2, a_3) = \gcd(\gcd(a_1, a_2), a_3).$$

   (c) (0.2p) Generalize the formula

   $$\mathrm{lcm}(a_1, a_2) = \frac{a_1 a_2}{\gcd(a_1, a_2)},$$

   to the case of finding $\mathrm{lcm}(a_1, a_2, \ldots, a_n)$. You must also prove that your formula is correct.

   The formula shall be such that a programmer who has a module for finding the greatest common divisor of any two integers, (but not for finding the least common divisor), can use your formula in some way, perhaps in a recursive way.