Matematiska Institutionen
KTH

Exam to the course Discrete Mathematics, SF2736, June 8, 2012, 08.00–13.00.

Observe:

1. You are not allowed to use anything else than pencils, rubber, rulers and papers at this exam.
2. To get the maximum number of points on a problem it is not sufficient to just give an answer, you must also provide explanations.
3. Bonus points from the homeworks will be added to the sum of the points on part I.
4. Grade limits: 13-14 points will give an Fx; 15-17 points will give an E; 18-21 points will give a D; 22-27 points will give a C; 28-31 points will give a B; 32-36 points will give an A.

Part I

1. (3p) Find all graphs G with the property that both G and its complement graph $\bar{G}$ are bipartite.
2. (3p) Find $3513^{561} \pmod{562}$.
3. (3p) A package shall contain seven items. You can choose among red balls in a box and nine distinct books from a book shelf. How many distinct such packages can you form if the package must contain at least one book and at least one red ball. The answer shall be given as an integer.
4. (3p) Find the number of ways to form a necklace consisting of 7 beads in the colors red, green and yellow.
5. (3p) Does there exist an abelian (commutative) group G with two distinct subgroups H and K with a pair of cosets that coincide, that is, there are two elements a and b of G such that $aH = bK$?

Part II

6. (3p) Show that every connected graph with more than two vertices, and containing the same number of vertices of valency (degree) one as there are vertices of valency three, must have at least one cycle.

7. (a) (1p) Prove that if n and m are integers such that $310n + 147m = 1$ then n and m must be coprime, that is, $\gcd(n, m) = 1$.

   (b) (3p) Are there three non-zero, and pairwise coprime, integers n, m and k such that
   $$310n + 217m + 147k = 1?$$

8. (4p) Eight boys and seven girls shall form three queues, that are labeled as queue no. 1, queue no. 2 and queue no 3. How many such distinct queues can you form if it is required that every queue must contain at least one boy and one girl, and the boys are placed either in the front or the rear of each queue?

Part III

9. Let $\mathcal{S}_n$ denote the group that consists of all permutations of the elements in the set $\{1, 2, \ldots, n\}$.

   (a) (2p) Show that $\mathcal{S}_4$ has exactly four subgroups of size 6.

   (b) (2p) Find the number of subgroups of $\mathcal{S}_5$ of size 6.

10. Suppose that G is a finite abelian group and that G_1, G_2, ..., G_n are subgroups of G satisfying
   $$G = \bigcup_{i=1}^{n} G_i \qquad \text{and} \qquad i \neq j \quad \Rightarrow \quad G_i \cap G_j = \{0\}, \tag{1}$$
   where 0 denotes the identity in G. Let C be the kernel of the map φ from $S = G_1 \times G_2 \times \cdots \times G_n$ to G defined by
   $$\varphi((g_1, g_2, \ldots, g_n)) = g_1 + g_2 + \cdots + g_n.$$

   (a) (2p) Show that we can define a distance function between the elements in S and an error-correcting procedure, in such a way that C is an 1-error-correcting code.

   (b) (2p) Does C have any further error-correcting properties?

   (c) (2p) Find an abelian group G of size 27 and a family of subgroups of G having the property in Equation (1) above.