**Problem session November 29, SF2736, fall 10.**

1. How many distinct necklaces with 10 beads can you form by using just black and white beads.

2. In how many ways can a cube be colored in three distinct colors, if the six squared sides shall be colored, the eight corners, or the 12 edges of the cube, respectively.

3. The matrix
$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$
is the parity check matrix of a 1-error correcting code $C$. Find all elements of $C$. Correct the word 011111. How many words cannot be corrected.

4. Find a linear 1-error correcting code $C$ of length 9, size $|C| = 32$ and containing the words 111100000 and 110000110.

5. Find a linear 1-error correcting code $C$ of size $|C| = 128$.

6. Find the maximum size of a 2-error correcting code of length 8.

7. Show that
$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \ldots + n\binom{n}{n} = 2^{n-1}n \ .$$

8. Solve the recursion
$$u_0 = 2 \ , \quad u_1 = -6 \ , \quad u_{n+2} + 8u_{n+1} - 9u_n = 8 \cdot 3^{n+1} \ , \qquad n \geq 1 \ .$$

9. Prove that the number of partitions of $n$ in which each part is 1 or 2 is equal to the number of partitions of $n + 3$ which have exactly two distinct parts.

10. Find the number of partitions of 16 in which each part is an odd prime.