Problem session November 1, SF2736, fall 10.

1. Solve

   (a) $13x + 18 = 13$ in the ring Z_{64}.

   (b)

   $$\begin{cases} 7x + 2y = 5 \\ 10x + 7y = 3 \end{cases}$$

   in the ring Z_{13}.

2. Find all elements b of Z_{17} such that the equation

   $$x^2 + 3x + b = 0 \ ,$$

   has a solution.

3. Find

   $$545^{112} \pmod{23} \qquad \text{and} \qquad 545^{112} \pmod{24} \ .$$

4. Show that if p is a prime number then

   $$(p-1)! \equiv -1 \pmod{p} \ .$$

5. Find all integers n and m such that $314n + 218m = 12$ with $0 \leq n \leq 20$.

6. (a) When is it true that

   $$\operatorname{lcm}(a, b, c) = \frac{a \cdot b \cdot c}{\gcd(a, b, c)} \ .$$

   (b) Find a formula for $\operatorname{lcm}(a, b, c)$.

7. Find the number of solutions of the equation $x^2 = 1$ in the ring Z_{990}.

8. Solve the equation $(x-5)(x+3) = 0$ in the ring Z_{56}.

9. (a) Assume that m, n and k are integers with $n \geq 2$ and $m \geq 2$, and assume that $n^2 = k \cdot m^2$. Prove that k is a square of an integer.

   (b) Assume that $\gcd(x, y) = 1$. Prove that if $xy = z^2$ for some integer z, then $x = n^2$ and $y = m^2$ for some integers n and m.

10. Show that if $2^n - 1$ is a prime number then n must be a prime number. Will the same statement be true if we substitute 2 by any integer $a \geq 2$.