Matematiska Institutionen
KTH

Exam to the course Discrete Mathematics, SF2736, at 14.00 to 19.00 on December 13, 2010.

Observe:

1. Nothing else than pencils, rubber, rulers and papers may be used.
2. Bonus points from the homeworks will be added to the sum of points on part I.
3. Grade limits: 13-14 points will give Fx; 15-17 points will give E; 18-21 points will give D; 22-27 points will give C; 28-31 points will give B; 32-36 points will give A.

Part I

1. (3p) Find the least positive remainder when 64^{128} is divided by 43.
2. (3p) Draw a graph with 10 vertices and 15 edges that contains a Hamiltonian cycle, but no Eulerian circuit.
3. (3p) Find the number of surjective maps f from the set $\{1, 2, 3, 4, 5, 6\}$ to the set $\{1, 2, 3, 4\}$ with the property that $f(1) \neq f(2)$.
4. (3p) Let G be the group $(Z_{13} \setminus \{0\}, \ \cdot\ )$. Find four non trivial subgroups of G. (You will get 2p if you find just three non trivial subgroups, and 1p if you just find one non trivial subgroup.)
5. (3p) Let p and q be any two odd integers. Show that 2^n divides

$$\sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} .$$

Part II

6. (3p) Show that every graph on 15 vertices, of which seven have degree (or valency) 3, four have degree 4, three have degree 5 and one has degree 6, must contain at least one cycle.

7. (4p) Let G be a cyclic group with an odd number of elements and with generator h. Let e denote the identity element of G. Assume that the element g of G satisfies

$$g^{314} = e \qquad \text{and} \qquad g^{416} = e \ .$$

   Is the above information sufficient to find the element g? If the answer is yes, find the element, otherwise explain why the information is not sufficient.

8. (4p) Consider the complete graph K_6 on six vertices which are colored with the colors red, green and blue. How many distinct graphs with colored vertices can you obtain from this colored K_6 by deleting two edges. .

Part III

9. A ternary code C of length n is a set of words of length n formed by using letters from the alphabet $\{0, 1, 2\}$. We define the distance between words of length n as the number of positions in which the words differ.

   (a) (2p) Show that the set of words in the code C, where

$$C = \{0000, 0111, 0222, 1012, 1120, 1201, 2021, 2102, 2210\} \ ,$$

   has the property that every possible ternary word of length 4 is at distance at most one from a unique word of C.

   (b) (4p) Find, and describe in a suitable way, another ternary code C of some length $n \geq 5$ that has the property that every possible ternary word of length n is at distance at most one from a unique word of C.

10. (4p) Let $\mathcal{S}_n$ denote the set of permutations on a set with n elements, and let p be a prime number less than or equal to n. Derive a formula for the number of solutions $\varphi \in \mathcal{S}_n$ to the equation

$$\varphi^p = \text{id}.$$