KTH Teknikvetenskap

SF2729 Groups and Rings
Midterm Exam
Saturday, April 17, 2010

Time: 09.00-11.00
Allowed aids: none
Examiner: Mats Boij

This midterm exam corresponds to Part I (groups part) of the final exam and the final grade will be based on the maximum of the results on this part in the midterm exam and in the final exam.

Each problem can give up to 6 points. In the first problem, you are guaranteed a minimum given by the result of the homework assignment. If you have at least 2 points from HW, you cannot get anything from Part a) of Problem 1, if you have at least 4 points from HW you cannot get anything from Part a) or Part b) of Problem 1.

The minimum requirements for the various grades are according to the following table:

Grade	**A**	**B**	**C**	**D**	**E**
Total credit	30	27	24	21	18
From Part I	13	12	11	9	8
From Part II	13	12	11	9	8

Present your solutions to the problems in a way such that arguments and calculations are easy to follow.

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(1) a) Show directly from the axioms that a group G in which $a * a = e$, for all elements a, has to be abelian. **(2)**

b) Find all subgroups of A_4 and write down the subgroup lattice. **(2)**

c) Show that if H and K are finite subgroups of a group G, we have that

$$|HK| = \frac{[H| \cdot |K|}{|H \cap K|},$$

where $HK = \{hk|h \in H, k \in K\}$. **(2)**

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(2) a) Define what a normal subgroup is and show that there is a well-defined group structure on the set of cosets of a normal subgroup H of a group G. **(2)**

b) Let G be the group of upper triangular matrices of the form

$$\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$$

in $\mathrm{Gl}_3(\mathbb{Z}_3)$. Determine the center $Z(G)$ and compute the factor group $G/Z(G)$. **(4)**

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(3) a) Let H and K be normal[1] subgroups of a group G such that $HK = G$ and $H \cap K = \{e\}$. Show that $G \cong H \times K$ **(2)**

b) The symmetric group S_4 can be presented by the generators $\{s_1, s_2, s_3\}$ and the relations $s_1^2 = s_2^2 = s_3^2 = (s_1 s_2)^3 = (s_1 s_3)^2 = (s_2 s_3)^3 = e$. Use this in order to determine the automorphism group of S_4. **(4)**

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[1]This was unfortunately not mentioned in original version of the exam.