Homework nr 6

Handed out November 27, 2007
To be handed in December 11, 2007

Part A Some complementary instruction for Homework5 B.
This is an exercise to tune the parameters. Adjust the scale vectors to the musical setting and plot with frequencies and time on marked on the axis

1. The complex Morlet wavelet was given by the formula[

$$\psi(x) = \frac{d}{dx} \exp\{-x^2\} \exp\{iAx\}$$

   where A is a positive number. The selection $A = 5$.
   might not give good enough frequency resolution

2. Find the sampling rate R of the signal, (see help wavread).

3. What will the frequency f when the scale is s and the oscillating term of the Morlet wavelet is $\exp\{iAx\}$

4. Adjustment to our music system there are 12 half-tone in an octave so us frequencies: $f_j = 2^{j/12} f_0$ or with quarter tones $f_j = 2^{j/24} f_0$, spanning two or three octaves. find the corresponding scales s_j, (which also depends on R and A). For a guitar the frequency 264 Hz to 1056 Hz would probably be a suitable choice.

5. experiment with different choices of the parameters A to get a suitable compromise between time and frequency precision.

Part B The local cosine transform
This assignment is rather new for me too, so I don't know how it will turn out. If you get problems with it don't hesitate to contact me
/Jan-Olov S

The data consists of N sample points. We assume the sampling is done at half points between integers: $n+\frac{1}{2}$. The sampling interval is broken up into intervals $[a_{j-1}, a_j]$ where a_j are integers.
Around each interval endpoint a_j, $0 < j < 2^k$ there is a transition interval $[a_j - b_j, a_j + b_j]$ where windows overlap. Here b_j is an integer and $b_j + b_{j-1} \leq a_j - a_{j-1}$ so that the transition intervals do not overlap. (For simplicity we may assume that $N = 2^m$ and there are 2^k intervals of lengths $L_j = a_j - a_{j-1} = L = 2^{(m-k)}$,where $0 < k < m$ are fixed and also that all b_j are equal: $b_j = b$)

1. Build a folding matrix (and its inverse: the unfolding matrix).
   This will be a matrix $\{c_{jk}\}$ of size $2b \times 2b$ with non-zero entries only on two crossing diagonals. There values are given by the 2×2 rotation matrices

$$\begin{pmatrix} c_{j,j} & c_{j,2b-j+1} \\ c_{2b-j+1,j} & c_{2b-j+1,2b+j-1} \end{pmatrix} = \begin{pmatrix} \cos\alpha_j & -\sin\alpha_j \\ \sin\alpha_j & \cos\alpha_j \end{pmatrix}$$
$$= \begin{pmatrix} \cos\alpha_j & -\cos\alpha_{2b+1-j} \\ \cos\alpha_{2b+1-j} & \cos\alpha_j \end{pmatrix}$$

   for $j = 1, \ldots, b$ where the rotation angles

$$\alpha_j = \frac{\pi}{4} P\left(\frac{j - b - \frac{1}{2}}{b}\right) \text{ for } j = 1, \ldots, 2b.$$

   We may chose $P(t) = t^3 - 3t$.

2. Folding the data in all the transition intervals. The matrix above applies to the data in the transition interval for each transition interval. Use this matrix to modify those data points which belongs to these transition intervals.

3. After this may organise the folded data (using reshape) into with 2^k columns$f_j()$ each of length 2^{m-k}.

   Since sampling points are at half integer points and also we looking at half integer frequencies the cosine transform will be

$$\begin{aligned} \hat{f}_j(k) &= \tfrac{2}{\sqrt{L}} \textstyle\sum_{n=1}^{L} f_j(n) \cos\{\pi(k - \tfrac{1}{2})(n - \tfrac{1}{2})/L\} \\ &= \text{Real part } \left[\tfrac{2}{\sqrt{L}} \textstyle\sum_{n=1}^{L} f_j(n) \exp\{i\pi(k - \tfrac{1}{2})(n - \tfrac{1}{2})/L\}\right] \end{aligned}$$

4. Compare the formula above with Matlabs fft by using 'help fft'. Find factors A_n and B_k so that the formula's will be consistent with $x(n) = A_n f(n)$ and $\hat{f}(k) = B_k X(k)$, where $x(n)$ and $X(k)$ are given by the formula 'help fft' in Matlab.

5. Try to Inverse the process by first taken the inverse of the cosine transform -by taking inverse fft with matlab involving the weight factors A_n and B_k above .
   Finally use the unfolding matrix to modify the data at the transition intervals.

6. Try the transform on the data guitar.wav. You may choose a subsequence of suitable length. Suggestion set $b = 8$ and $L = 64$.