

## Inlämningsuppgift nr 1

Utgiven den 4 september, 2007

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### Hand calculation of the expansion in the Haar system $\{h_{kj}\}$ ,

The Haar System of function is given by

$$h_{kj} = 2^{j/2} h(2^j x - k),$$

where

$$h(x) = \begin{cases} -1 & \text{for } 0 \leq x < \frac{1}{2}. \\ 1 & \text{for } \frac{1}{2} < x \leq 1. \end{cases}$$

Given is the following function  $f$  on the interval  $I_0 = [0, 1]$ .

$$f(x) = \begin{cases} x & \text{for } 0 \leq x \leq \frac{9}{16}, \\ \frac{1}{4} & \text{for } \frac{9}{16} < x \leq 1. \end{cases}$$

In the following  $N = 4$ . The drawing of the graphs below could be done in the same diagram if using different colors.

1. Draw the graph of  $f$ .
2. Divide the interval  $I_0$  into  $2^N$  equally size intervals. Do the projection  $f$  into a step-wise constant function  $f_0$  on those intervals. (Minimizing the norm  $\|f - f_0\|$ ). Draw the graph of  $f_0$ .
3. Make a Haar wavelet expansion of  $f_0$  using functions  $h_{kj}$ ,  $j = N - 1, \dots, 0$  were and, the characteristic function  $\chi_{I_0}$  of the unit interval. List the corresponding coefficients. (Dont expand  $\sqrt{2}$  numerically)
4. Verify that  $f_0$  can be exactly reconstructed from those wavelet coefficients.
5. Replace the smallest 8 coefficients by zero and make an approximative reconstruction  $f_A$  from the remaining 8 coefficients. Draw the graph of  $f_A$ .
6. Error estimation: Estimate the the  $L^2$  norms  $\|f\|, \|f - f_0\|, \|f_0 - f_A\|$  and  $\|f - f_A\|$ .
7. Use a hand calculator to compute the signal-to-noise ratio

$$SNR = -20 \cdot {}_{10} \log \frac{\|f - f_A\|}{\|f\|}$$