KTH Matematik

Homework 1
Mathematical Systems Theory, SF2832
Fall 2014
You may use min(5,(your score)/4) as bonus credit on the exam.

1. Find the state transition matrix $\Phi(t,s)$ for the following systems

(a) $\dot{x}(t) = \begin{bmatrix} 1 & \sin(t) \\ 0 & 1 \end{bmatrix} x(t)$
.... (2p)

(b) $\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} x(t).$
.... (3p)

2. (a) Let

$$\dot{x} = A(t)x$$

and assume $A^T(t) = -A(t)$. Show $\Phi^T(t,s) = \Phi^{-1}(t,s)$. .... (2p)

(b) Let

$$\dot{x} = A(t)x.$$

Show that if $\int_s^t A(\tau)d\tau$ and $A(t)$ commute for all t, s, then the state transition matrix $\Phi(t,s) = exp(\int_s^t A(\tau)d\tau)$. .... (3p)

3. Consider

$$\begin{aligned} \dot{x} &= Ax, \ x \in R^n \\ y &= Cx, \ y \in R^p \\ x(0) &= x_0 \end{aligned}$$

where A and C are constant matrices.

(a) Show that if $x(0) \in ker\ \Omega$, then $x(t) \in ker\ \Omega$, $\forall t \geq 0$, where $\Omega = (C^T, A^TC^T, \cdots, (A^{n-1})^TC^T)^T$. (3p)

(b) Show that the above system is observable if and only if the only solution that satisfies $Cx(t) = 0, \forall t \geq 0$ is $x(t) = 0$. .... (2p)

4. The following is linearized model of a so-called inverted double pendulum

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -a_1 & 0 & -a_1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & a_2 & 0 & -a_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 3a_3 & 0 & -a_4 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ b_1 \\ 0 \\ -15b_2 \\ 0 \\ -b_2 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and all the parameters are positive.

(a) Check controllability for this system. Can we find at least one set of parameters a_i, b_i such that the system is controllable? (you can use, for example, Maple to help). .... (3p)

(b) Is the system observable? .... (2p)