Risk management in finance: an application of extreme value theory

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The Value–at–Risk (VaR) defines the risk of a portfolio as the 5%–quantile of the profit–loss distribution. Two standard estimation methods are the empirical quantile and the normal quantile. Both methods have frequently (and quite correctly) been critisized.

Under the achronym *let the tails speak for themselves*, Richard Smith and his collaborators developed methods, which only used such observations for the quantile estimation, which are responsible for extremal behaviour. For the VaR these are the largest negative values. For independent observations this has meanwhile become a standard application of well-known methods. We estimate the VaR with different methods and explain the pros and cons of the outcome ([4]).

Most financial data are, however, not iid but show some complex dependence structure. Such data are then modelled by diffusion processes or (G)ARCH models. Figure 1 shows the simulated path of the Cox-Ingersoll-Ross term structure model. We describe the extremal behaviour of diffusion models ([2] and, for estimation methods, [5]) and (G)ARCH-models ([1] or [3], Section 8.4.).

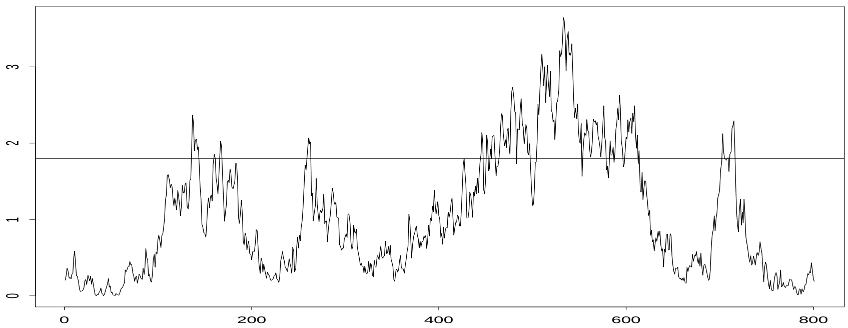

Figure: Simulated sample path of the Cox-Ingersoll-Ross model, given as solution to the SDE

$$dX_t = (c - dX_t)\,dt + \sigma\sqrt{X_t}\,dB_t\,, t > 0\,,$$

with (B_t) standard Brownian motion und parameters $c = d = \sigma = 1$.

References

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